ARC CONSISTENCY WITH NEGATIVE VARIANT TABLES

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Disclaimer:
All work on my papers “Arc Consistency with Negative Variant Table” and “Column Oriented Compilation of Variant Tables” was performed privately during the last two years after transition into partial retirement.

It and the accompanying implementation are neither endorsed by SAP nor do they reflect ongoing SAP development.

Notwithstanding:
The motivation for this work lies in my past at SAP and is based on insights and experiences with the SAP product configurators. The terminology follows that used in conjunction with the SAP Variant Configurator and originates from roots in the manufacture of „variants“.

See paper(s) for references

Negative Variant Table (NVTAB) $\mathcal{U}$

- List of exclusions/ disallowed combinations of product properties in tabular form

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Color	Size	Print
Black	Small	STW (Save the Whales)
Red	Small	MIB (Men in Black)
Red	Medium	MIB
Red	Large	MIB
Red	Small	STW
White	Small	MIB
White	Medium	MIB
White	Large	MIB
White	Small	STW
Blue	Small	MIB
Blue	Medium	MIB
Blue	Large	MIB
Blue	Small	STW

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- Same example: t-shirt
- Customer requirement

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White	Large	MIB
White	Small	STW
Blue	Small	MIB
Blue	Medium	MIB
Blue	Large	MIB
Blue	Small	STW

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Customer motivation for using NVTABs:

- Negative representation more compact
- Global domains may not be finite
- <u>Independence of changes to global domains</u>
  - Value domains may change often (typically daily)
  - Hope that NVTABs are less sensitive to these changes

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Arc-consistency (constraint propagation): Eliminate all values that do not have support from each column domain

Use of NVTABs in Configuration

Arc-consistency (constraint propagation): Eliminate all values that do not have support from each column domain

➔ Here, I deal with arc-consistency only

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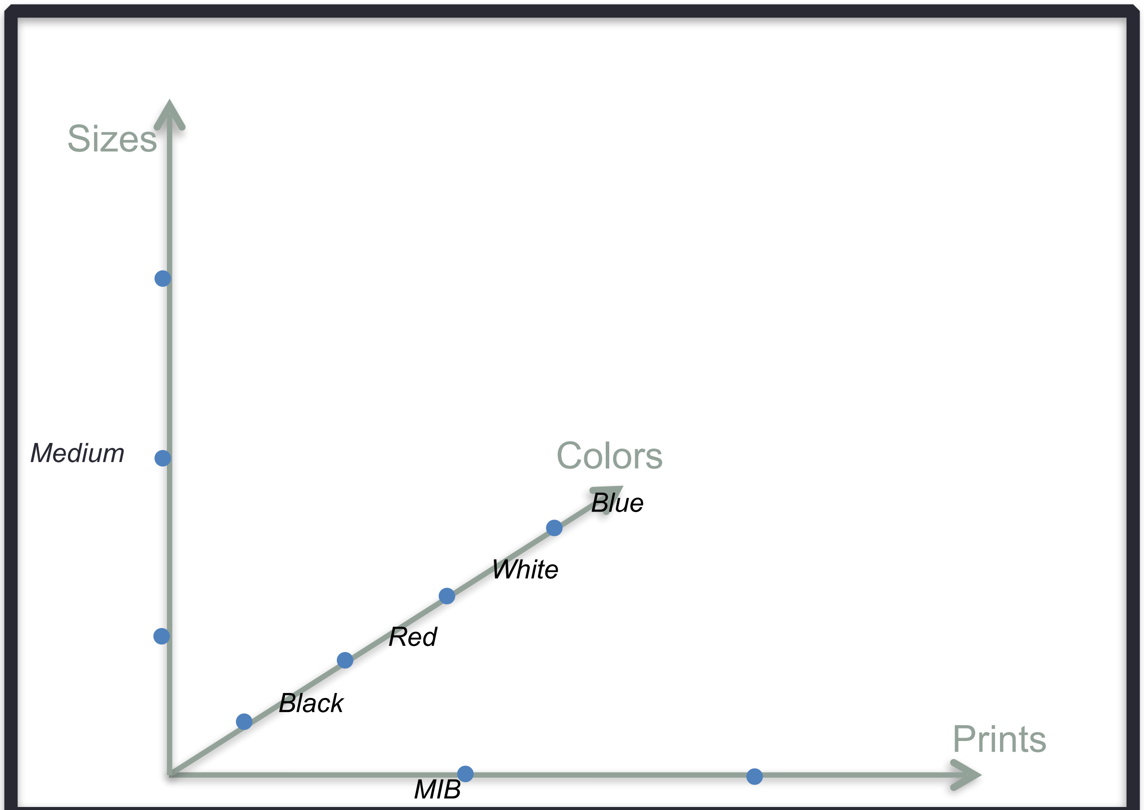

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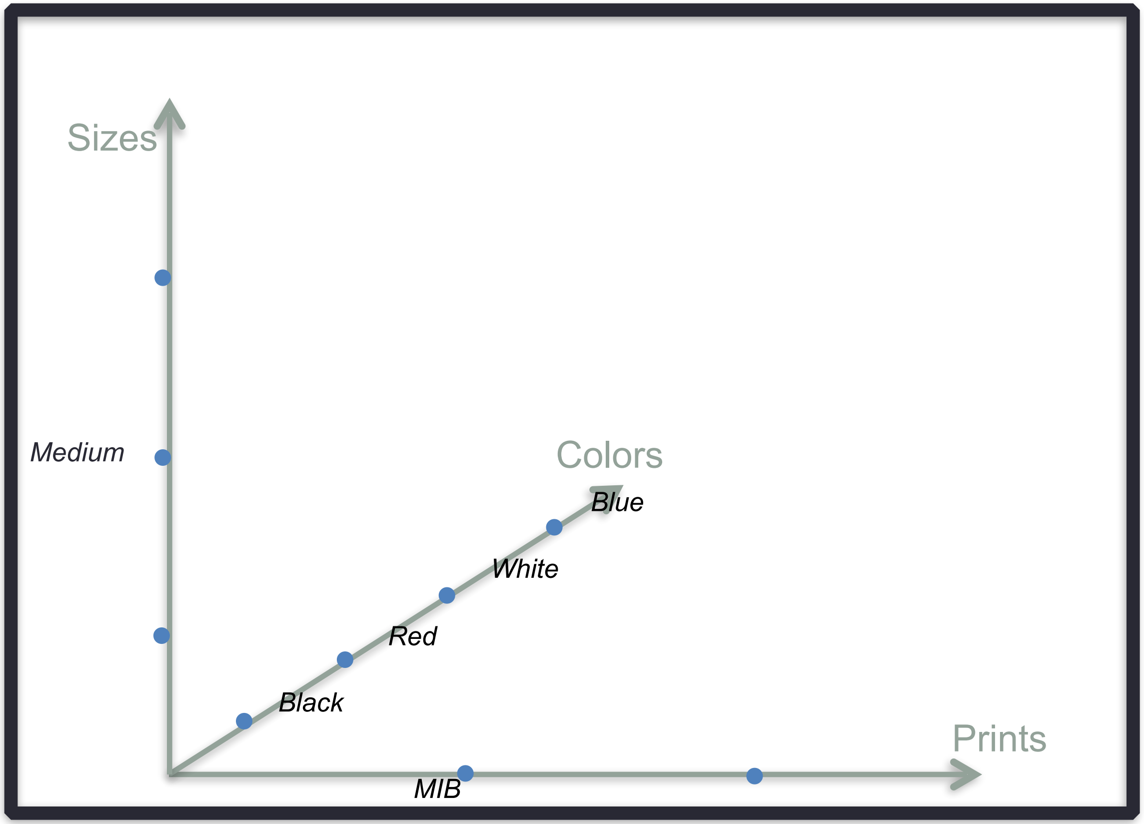
  - A VTAB/NVTAB is a (finite) subset of the “solution space”



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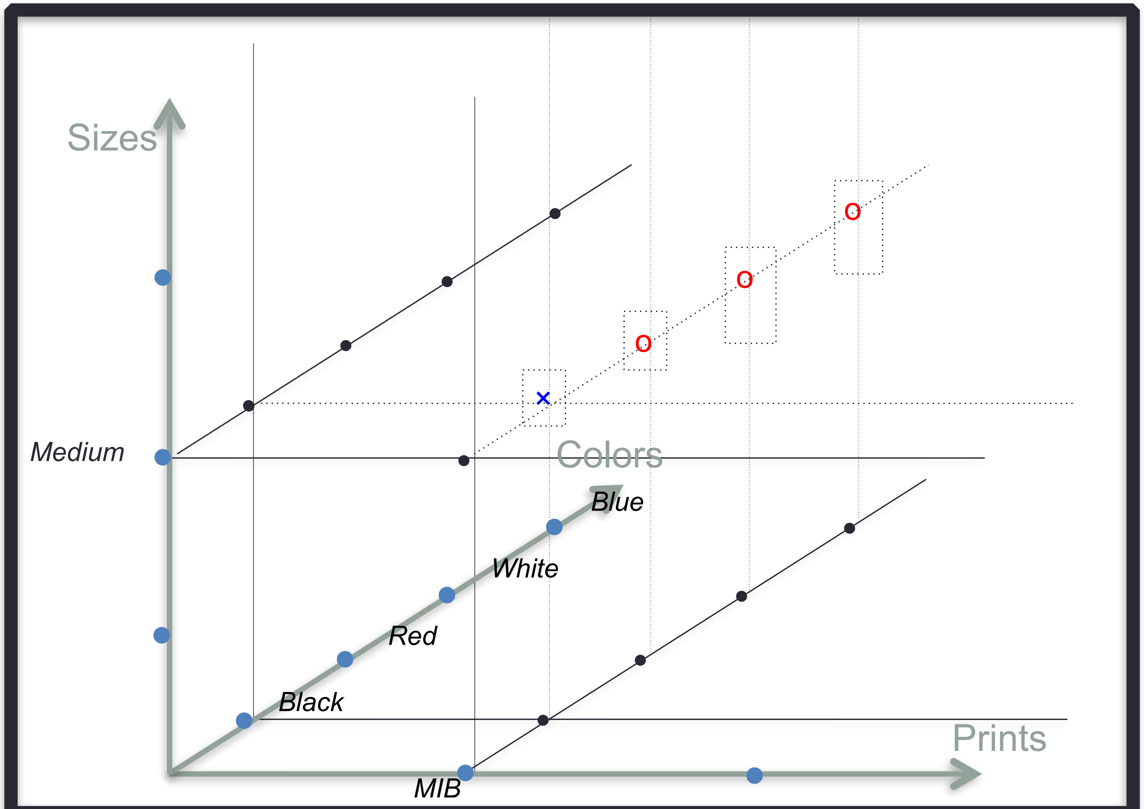


Legend
x – row in table
o – row not in table,

	Black	Red	White	Blue
Large/MIB	*x*	*o*	*o*	*o*
Large/STW	*x*	*x*	*x*	*x*
Medium/MIB	*x*	*o*	*o*	*o*
Medium/STW	*x*	*x*	*x*	*x*
Small/MIB	*x*	*o*	*o*	*o*
Small/STW	*o*	*o*	*o*	*o*

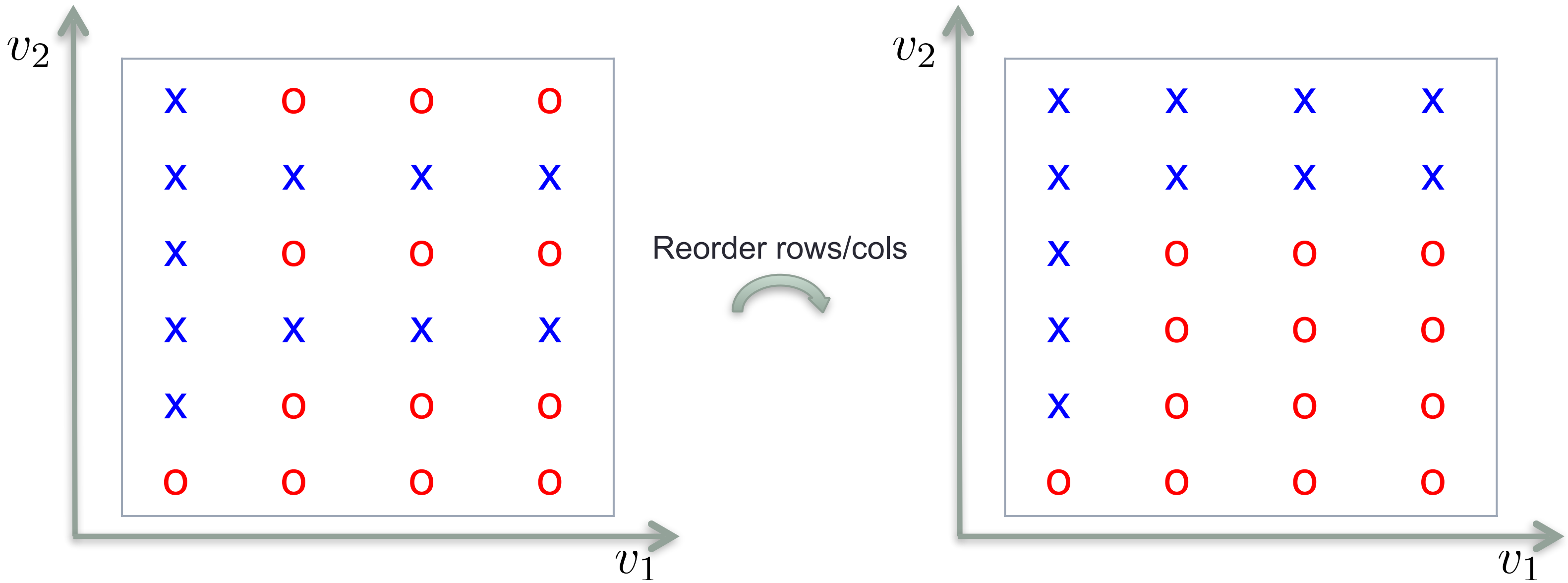
Two-dimensional depiction of valid t-shirts

Solution space: (2x3) x 4 = 24 rows

Legend

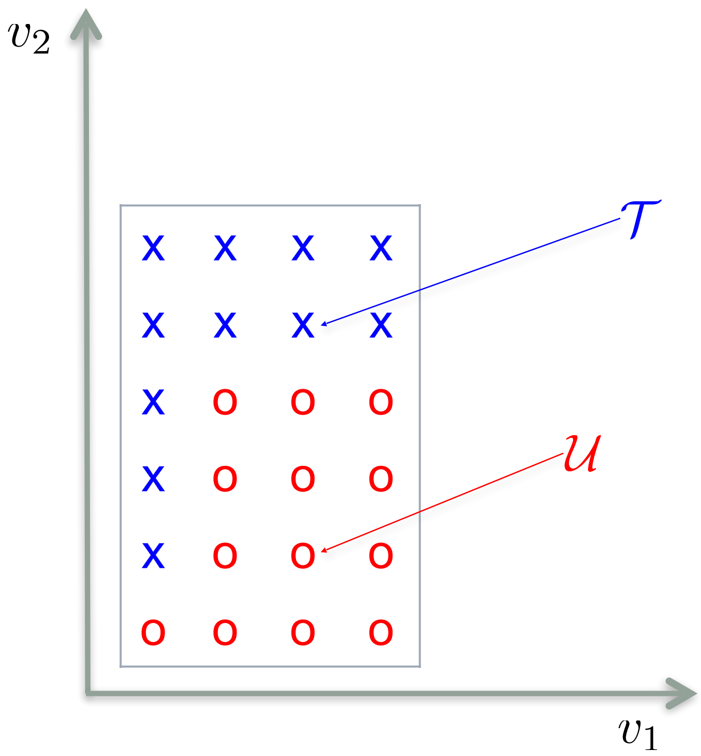
x – row in table

o – row not in table

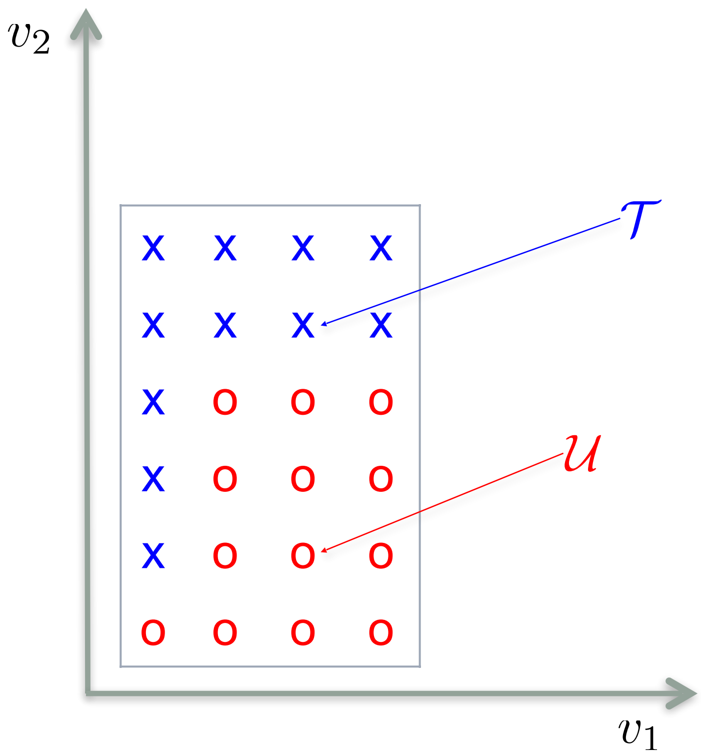


Abstract depiction of t-shirt VTAB/NVTAB and reordering

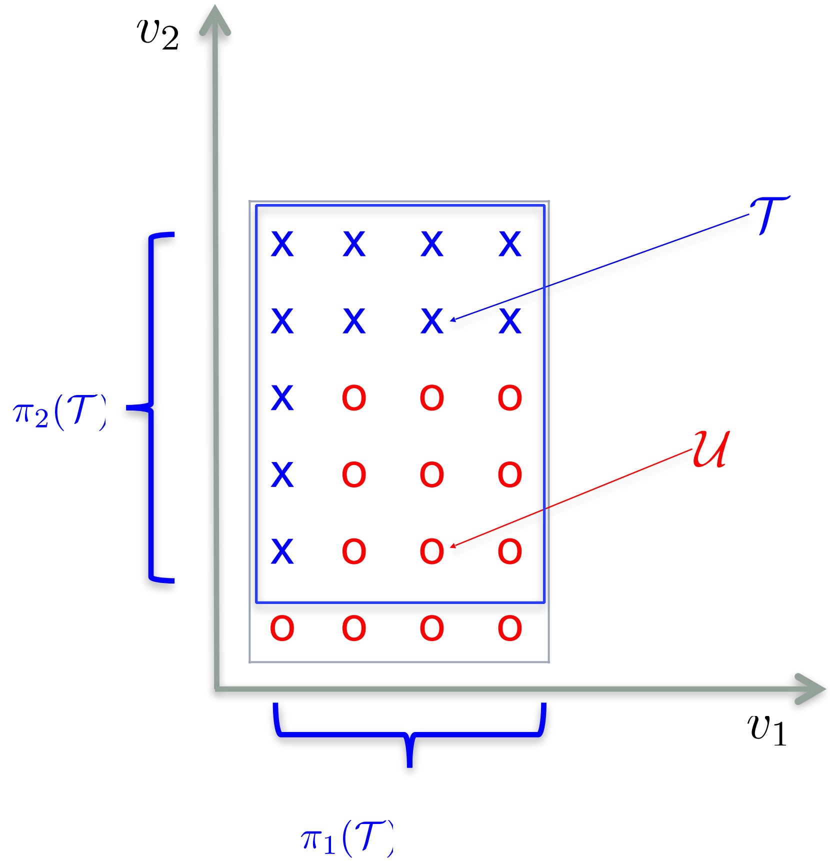
For VTAB everything outside depicted area is “x” (allowed combination)
For VTAB everything outside depicted area is “o” (dis-allowed combination)



v_2
$\mathcal{T}$
x x x x
x x x x
x o o o
x o o o
x o o o
o o o o
$\mathcal{U}$
v_1



Projections onto the *k*-axes and supported values



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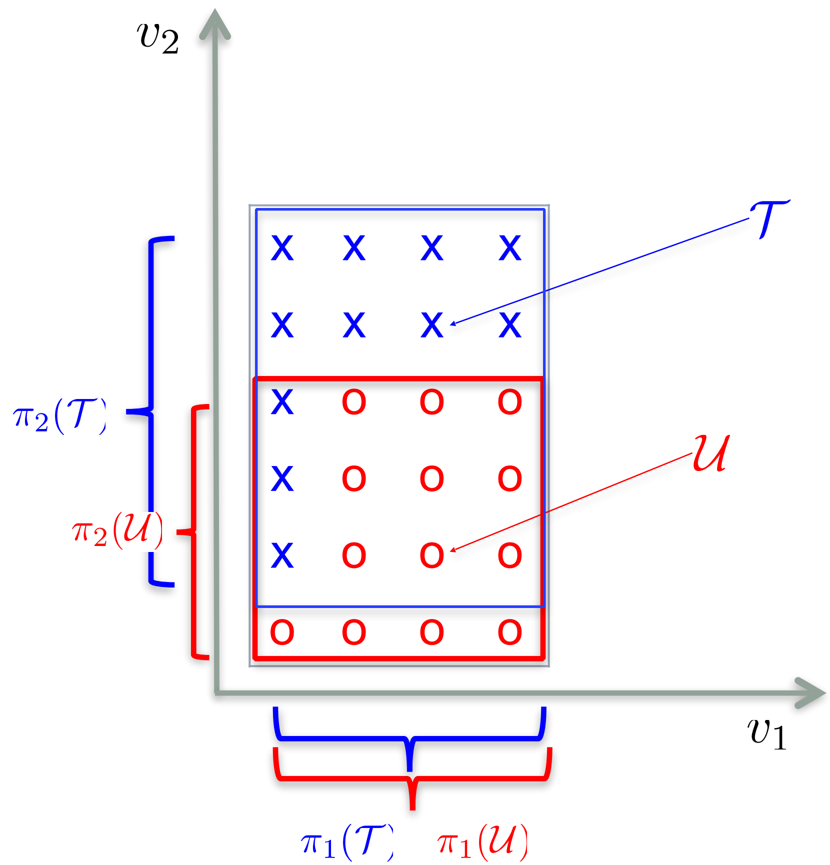

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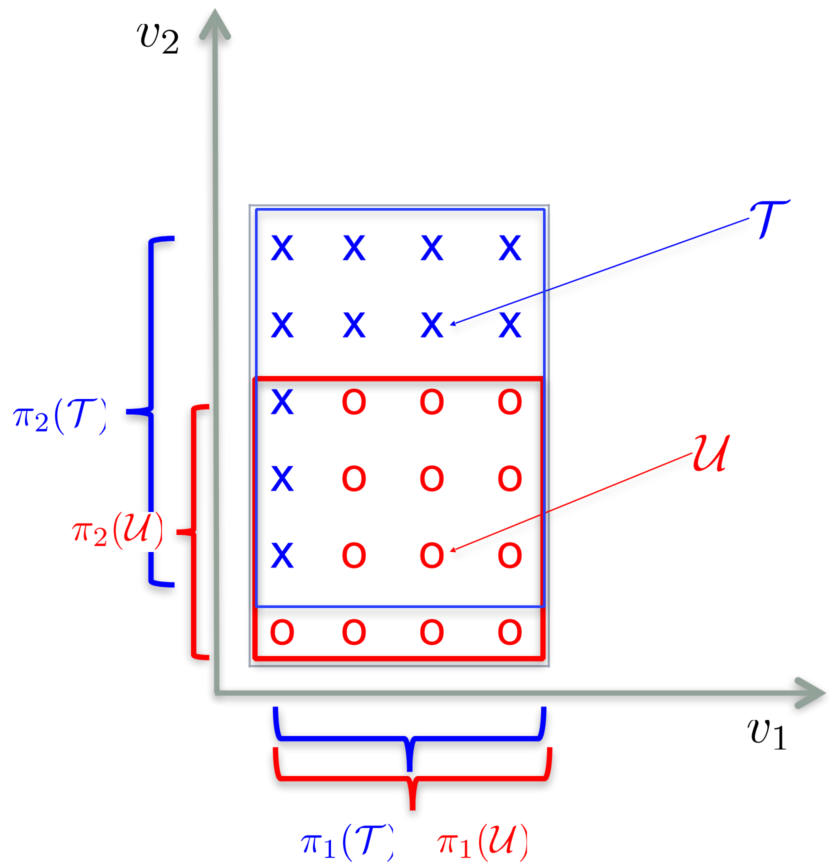


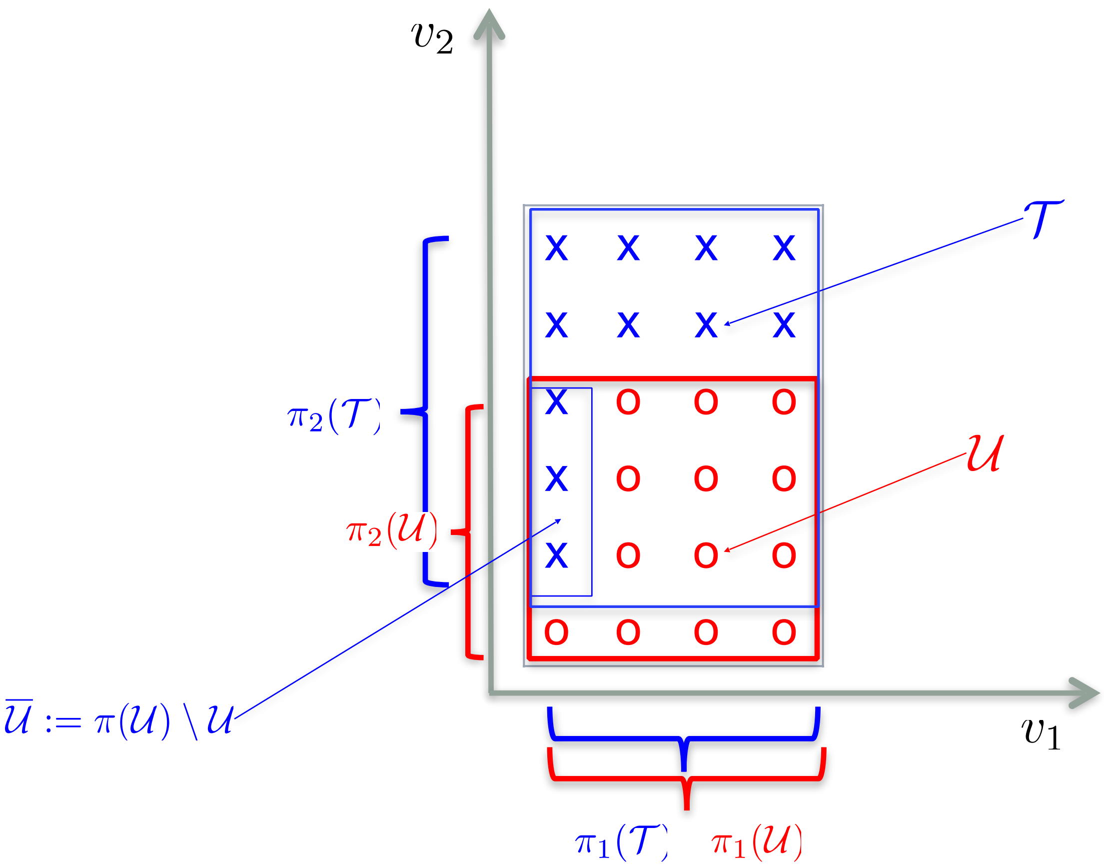
Projections onto the *k*-axes and supported values

$\pi_j(\mathcal{T})$ – projection of valid rows onto *j*-th axis

$\pi_j(\mathcal{U})$ – projection of invalid rows onto *j*-th axis

A value in $\pi_j(\mathcal{T})$ has support (is arc-consistent)

$\bar{\mathcal{U}} = \pi(\mathcal{U}) \backslash \mathcal{U}$ is positive VTAB



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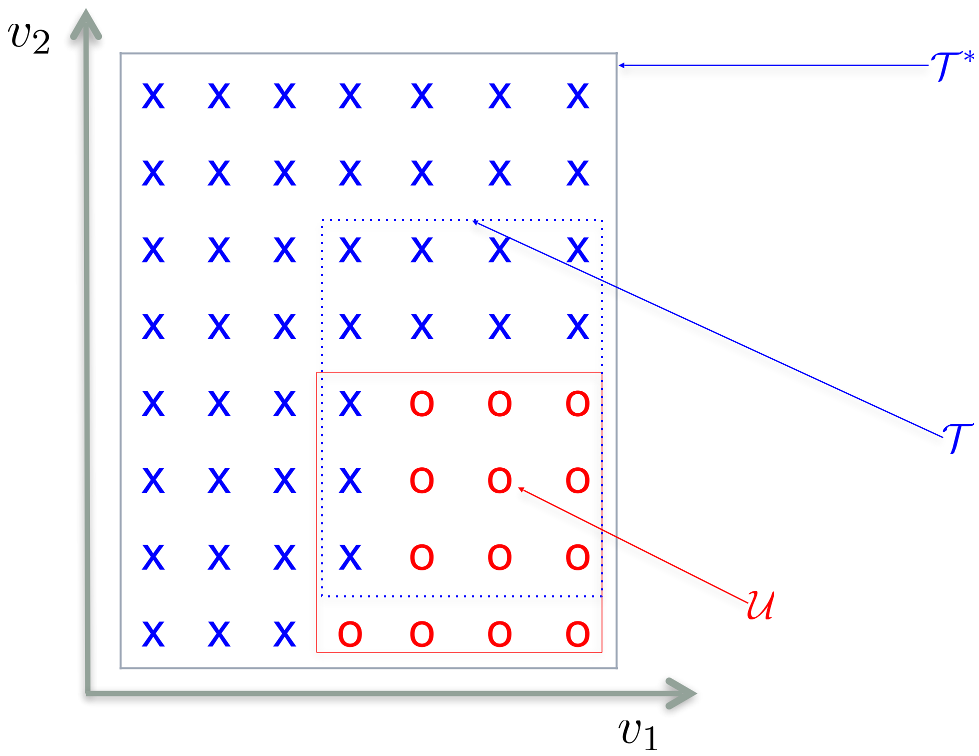
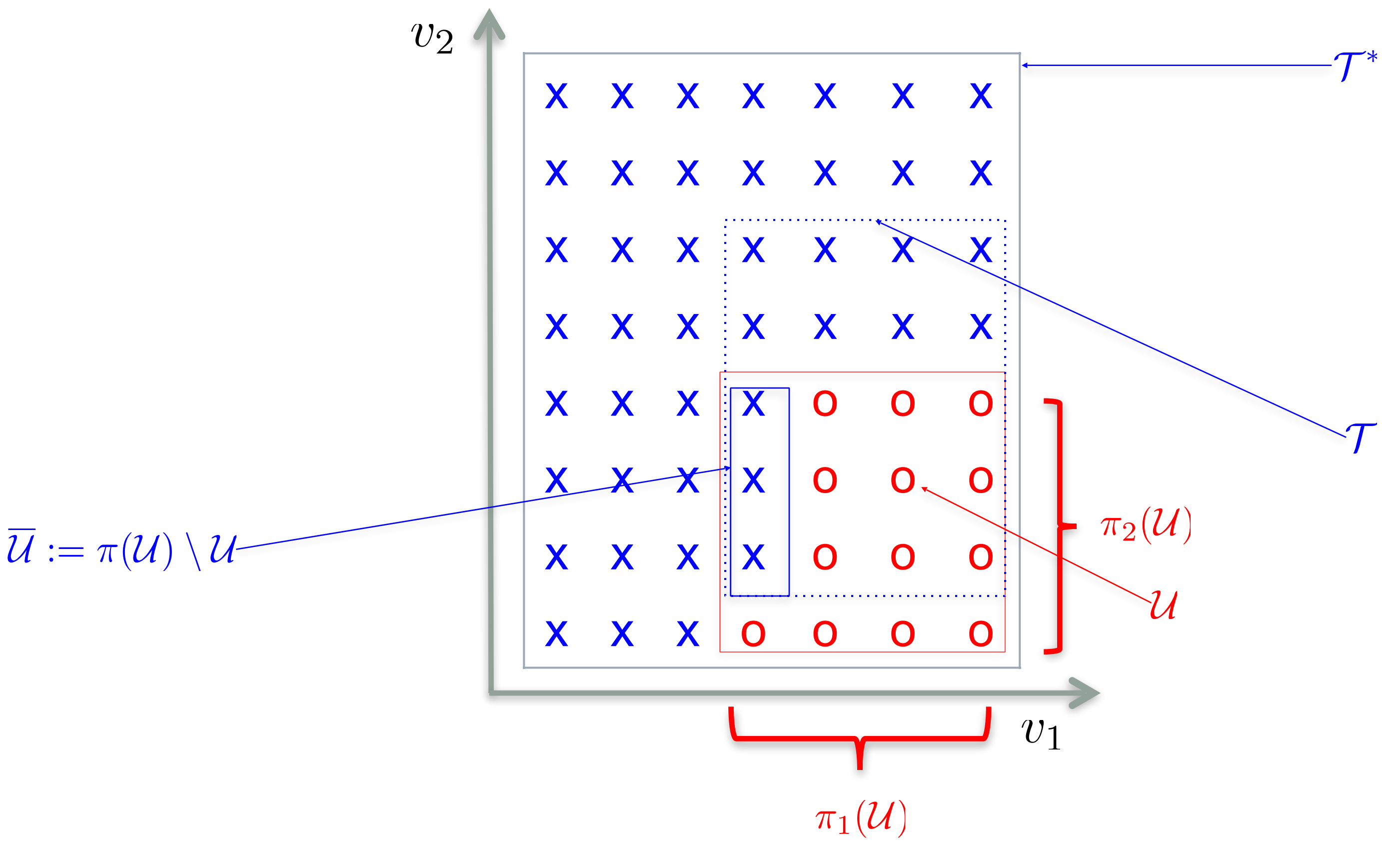


$\mathcal{T}^*$: Adding more values to v_1 and v_2 while leaving NVTAB $\mathcal{U}$ unchanged



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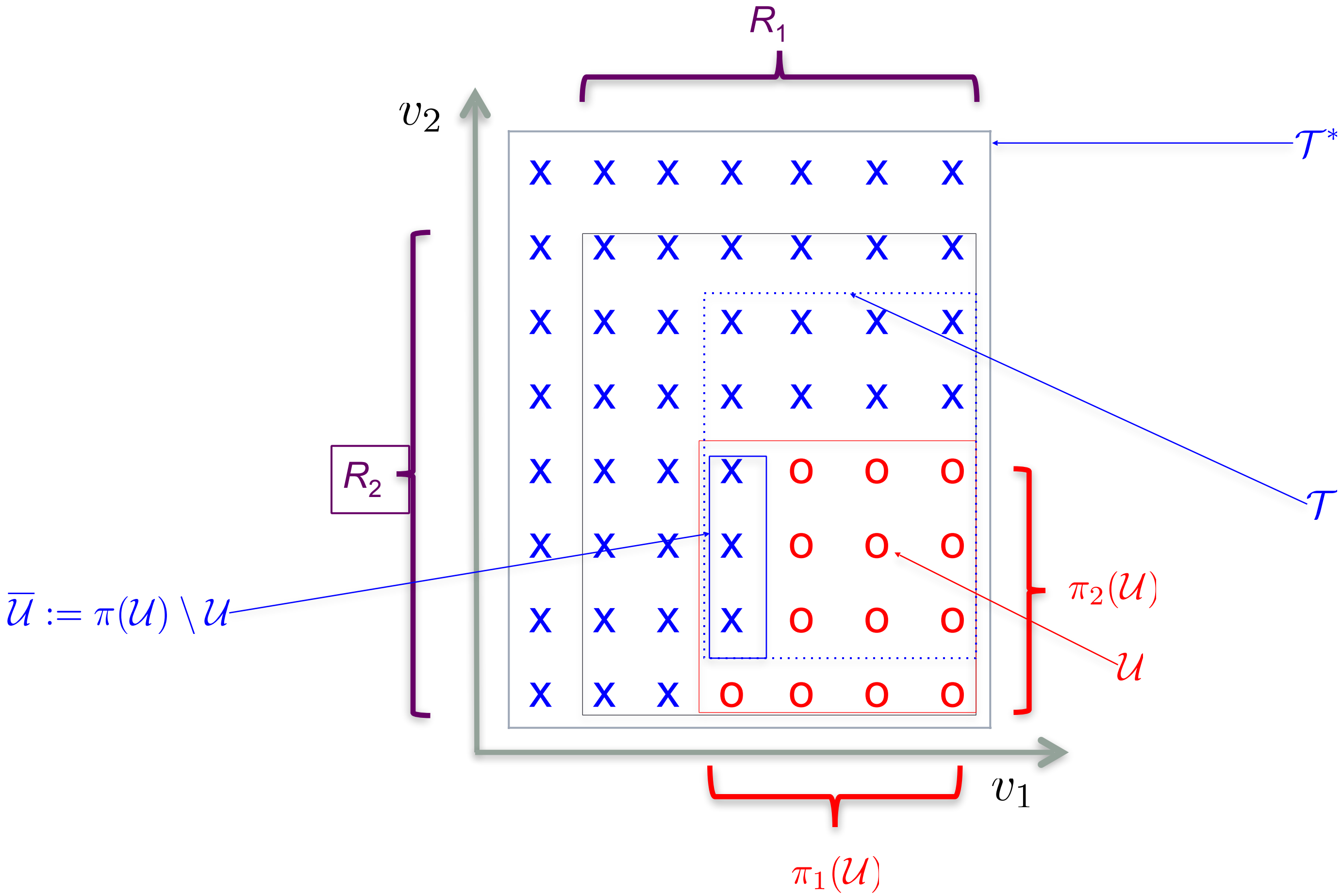

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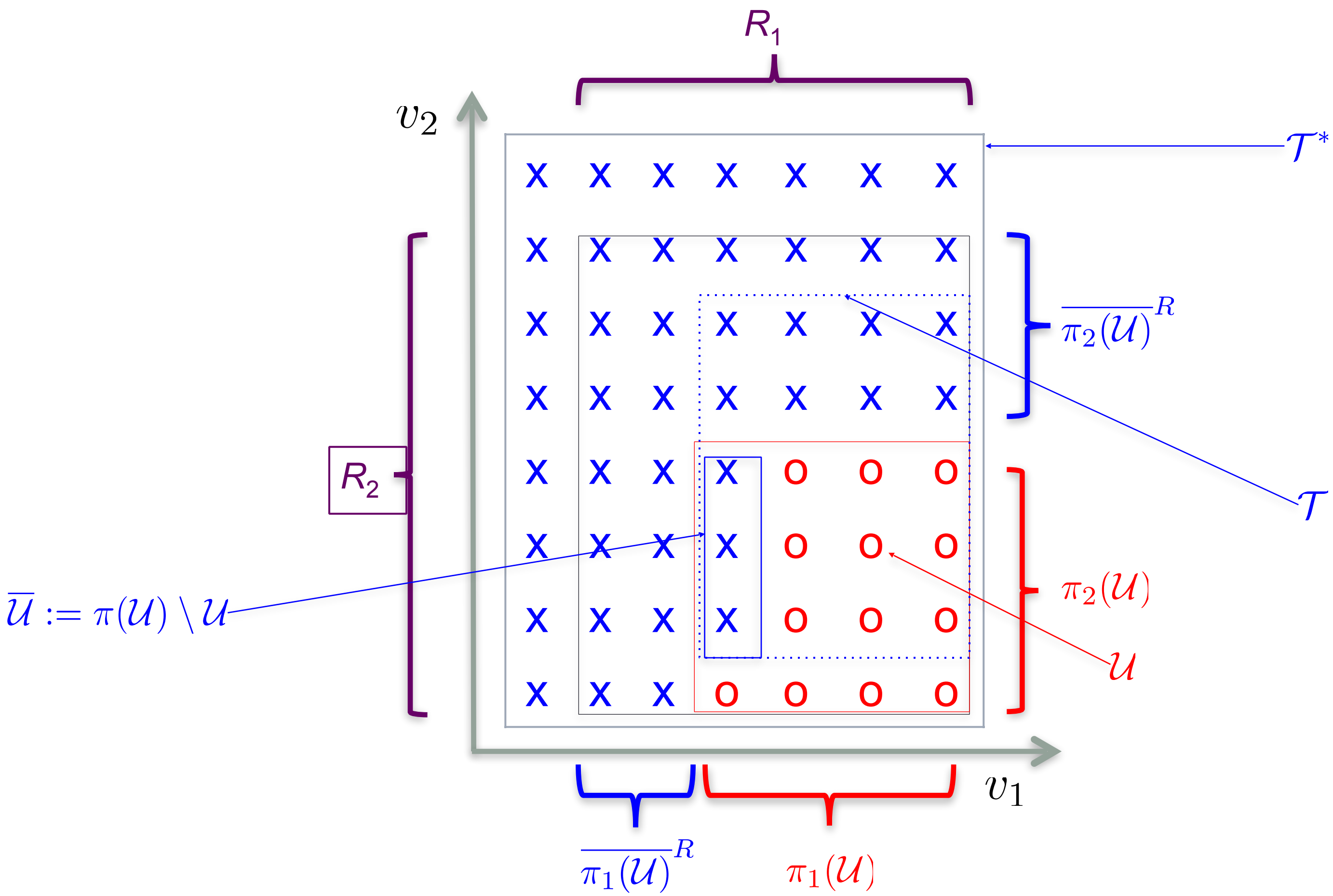

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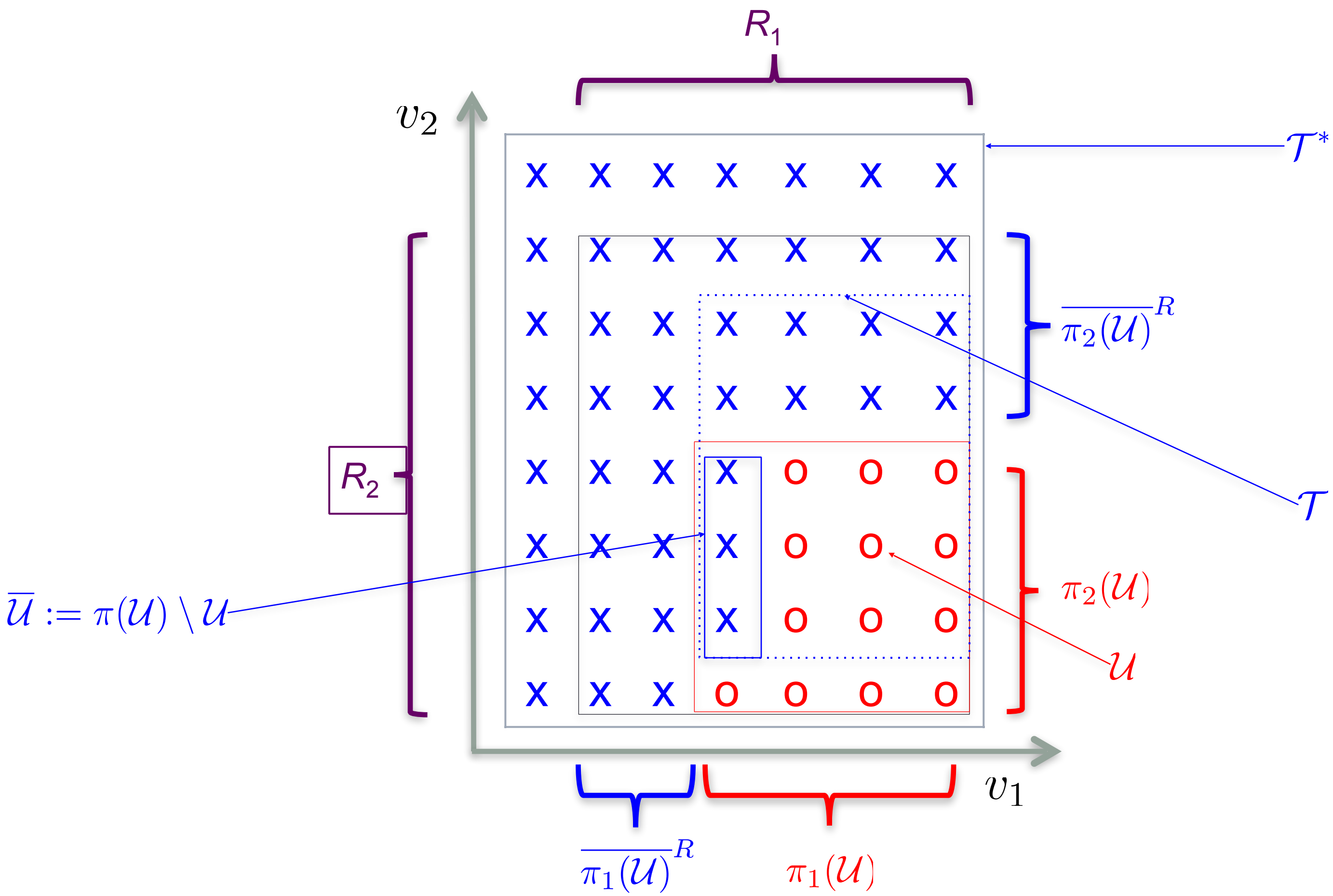

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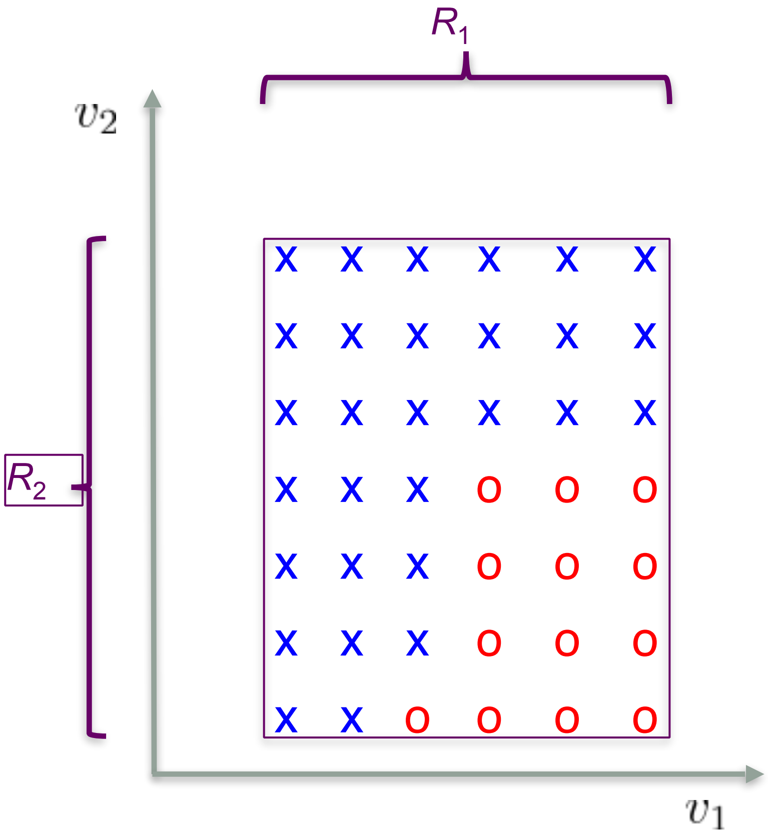
$\pi_j(\mathcal{U})$ – projection of NVTAB $\mathcal{U}$ onto j-th axis (determined at maintenance time)

R_j – restrictions known at run-time (time of evaluation)

$\overline{\pi_j(\mathcal{U})}^R$ – complement of $\pi_j(\mathcal{U})$ to R (determined at run-time)

Obs#1 – Decomposition of Solution Set



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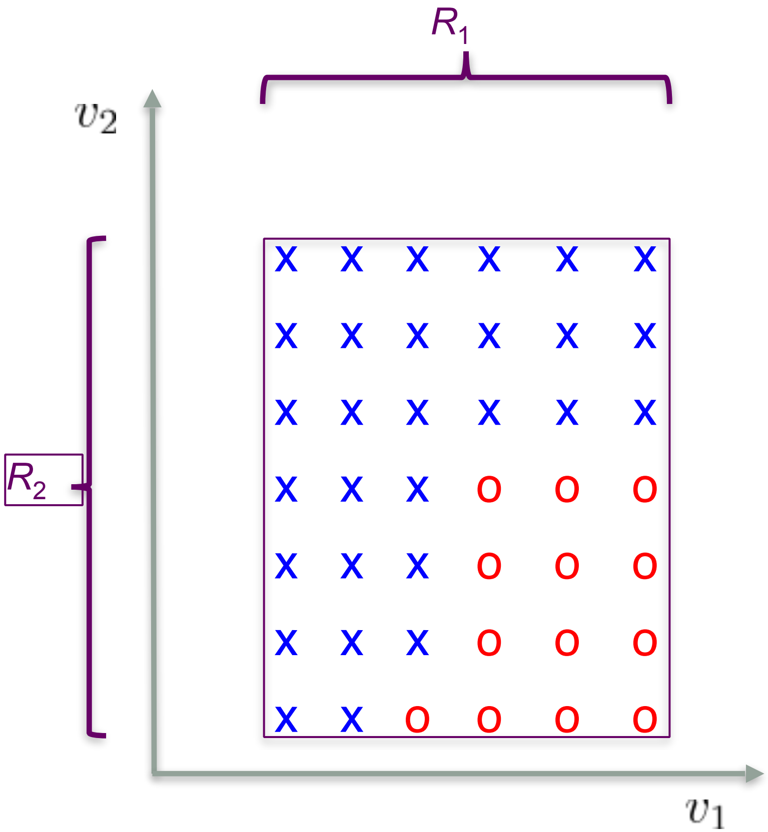

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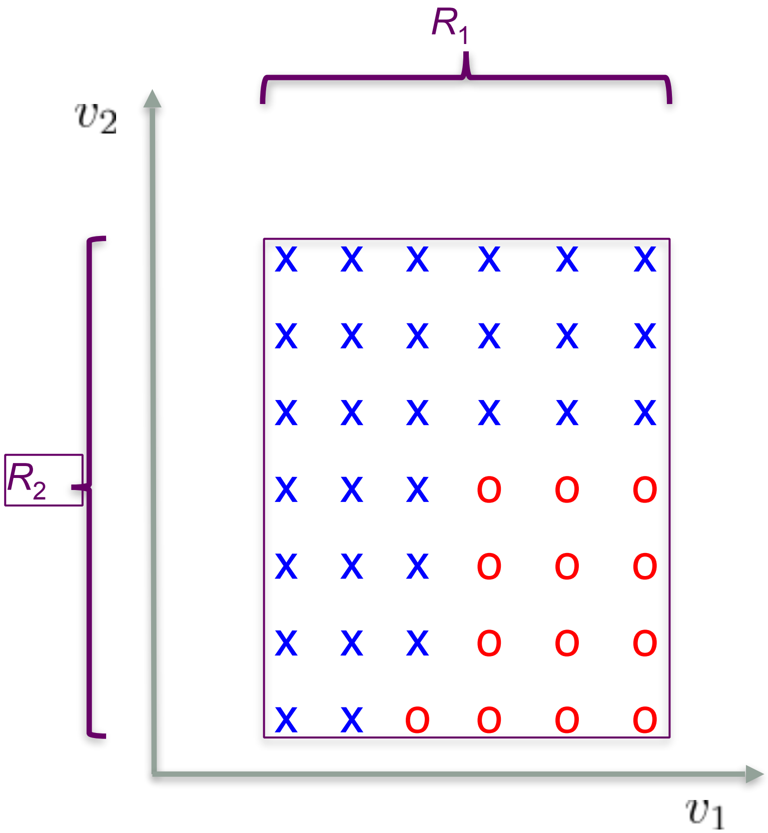

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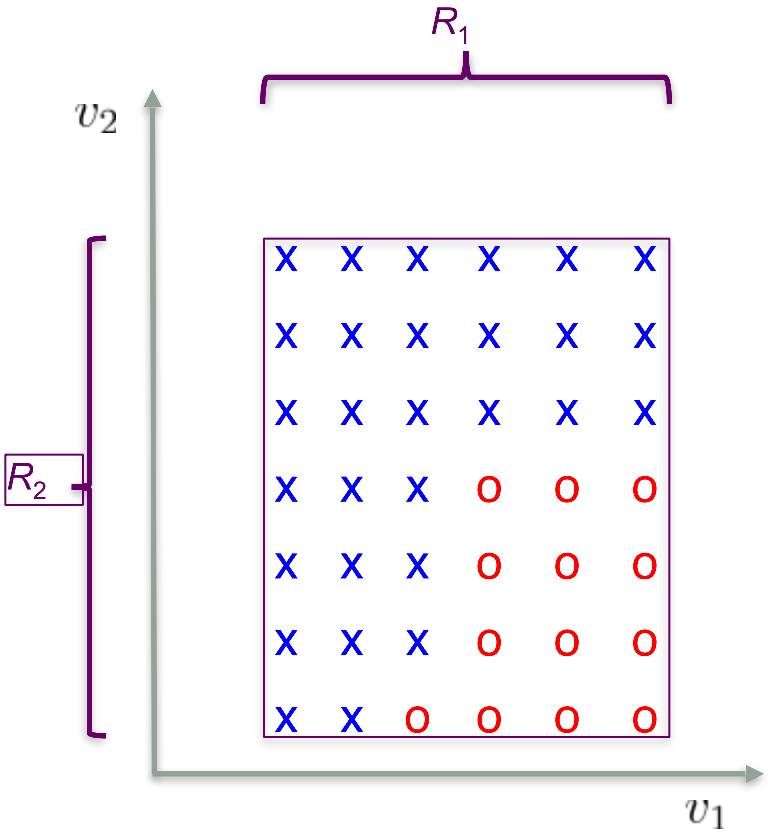

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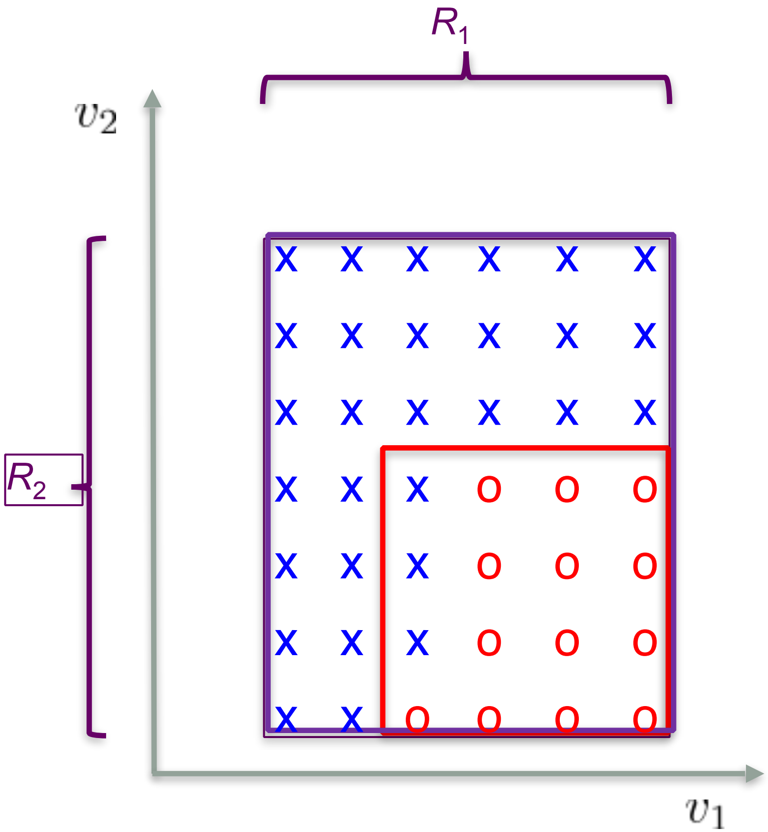

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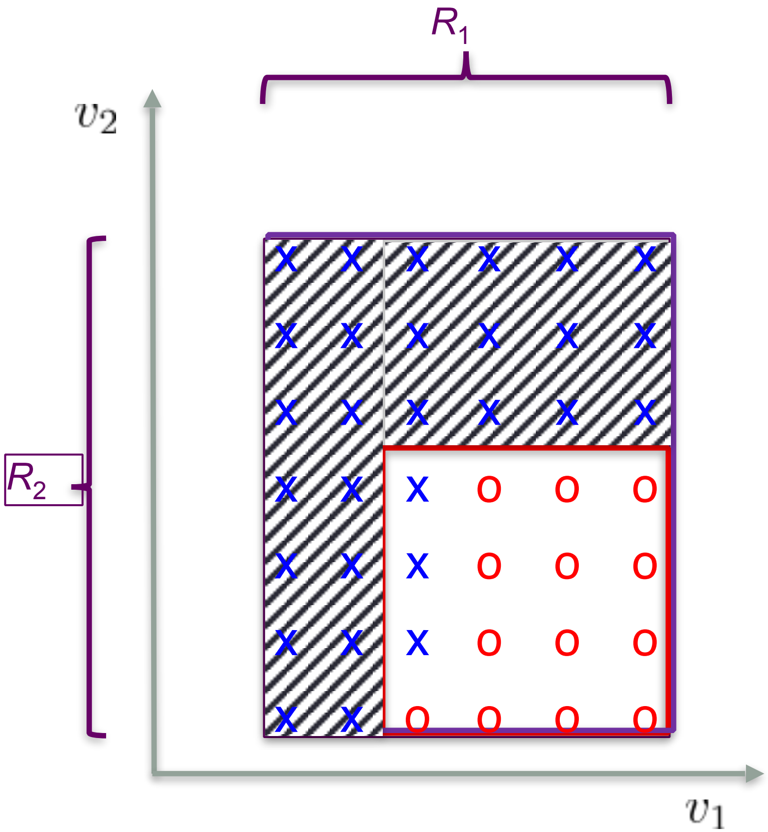

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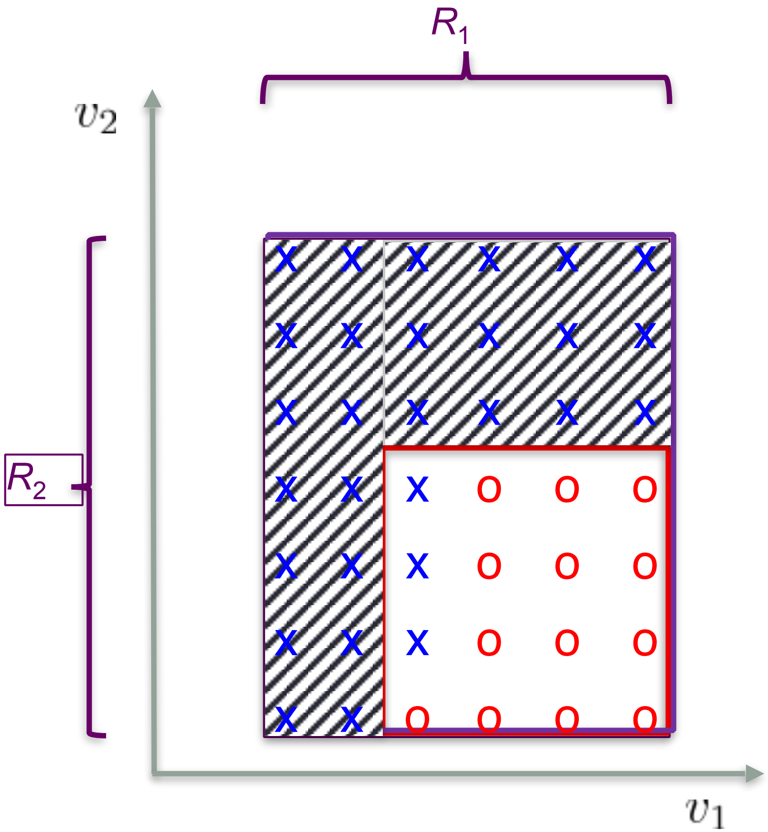

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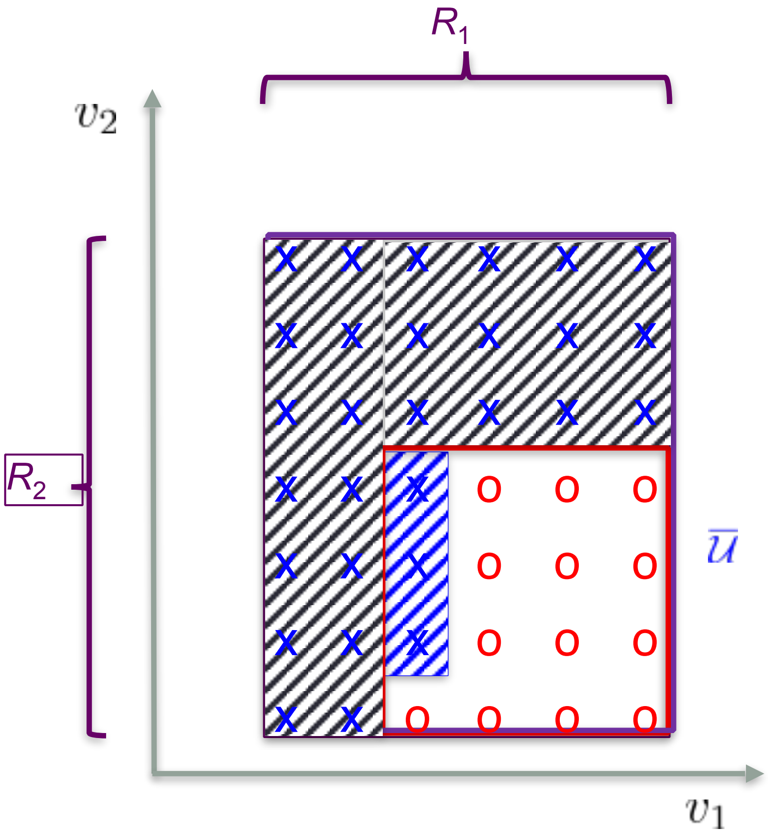

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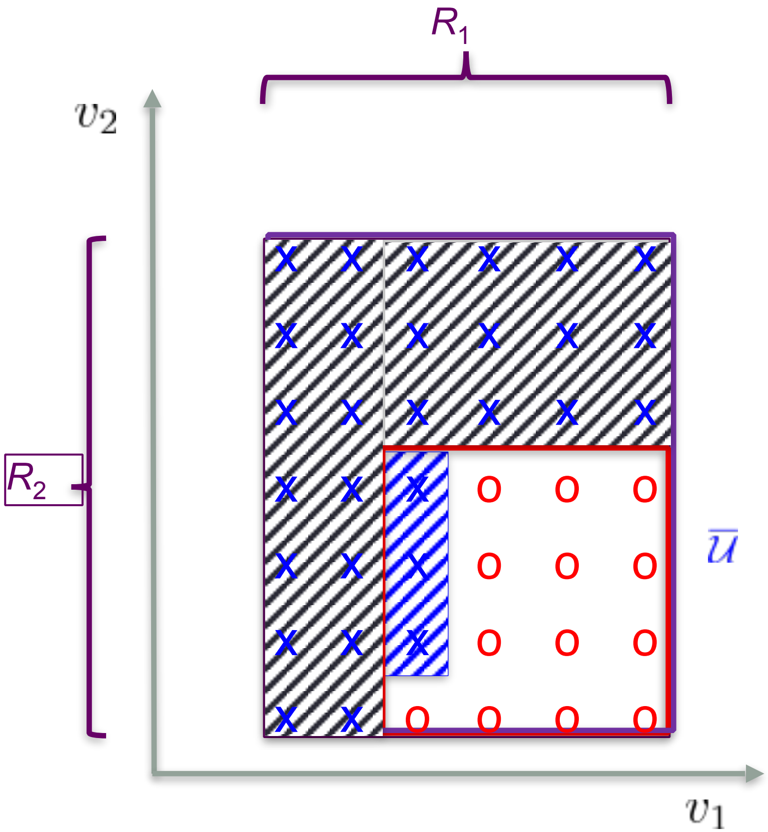

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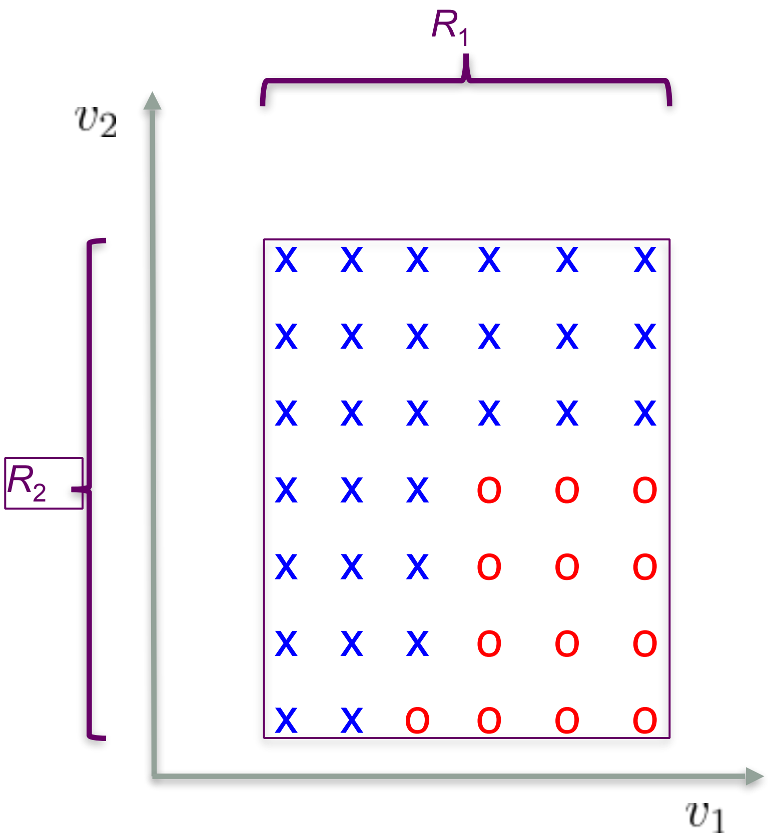
$$\mathcal{S}^{\mathcal{U},R} := (R \setminus \pi(\mathcal{U})) \cup (\overline{\mathcal{U}} \cap R)$$

Disjoint Decomposition of $R \setminus \pi(\mathcal{U})$



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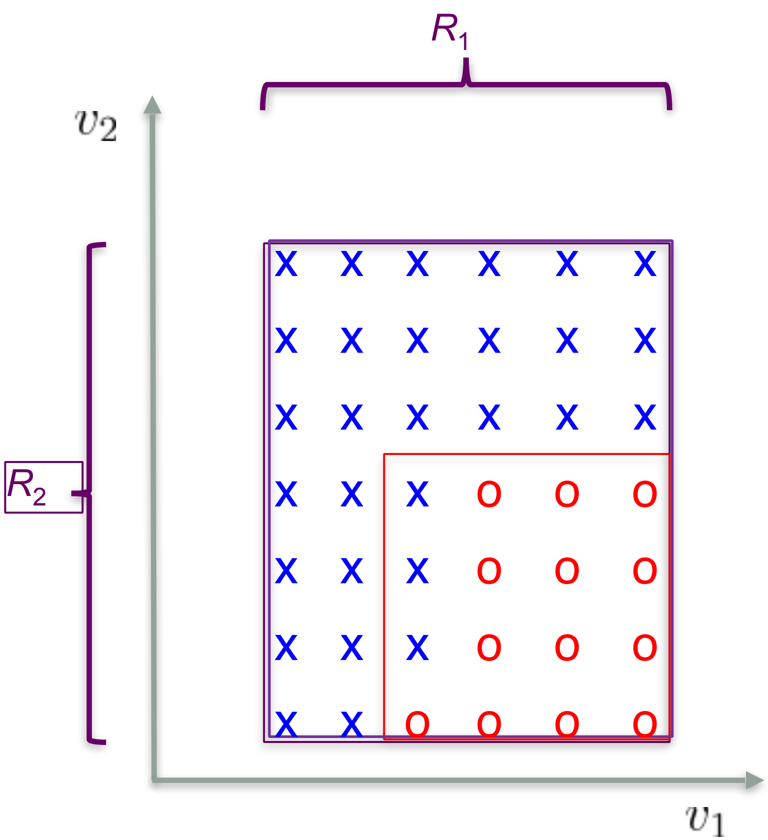

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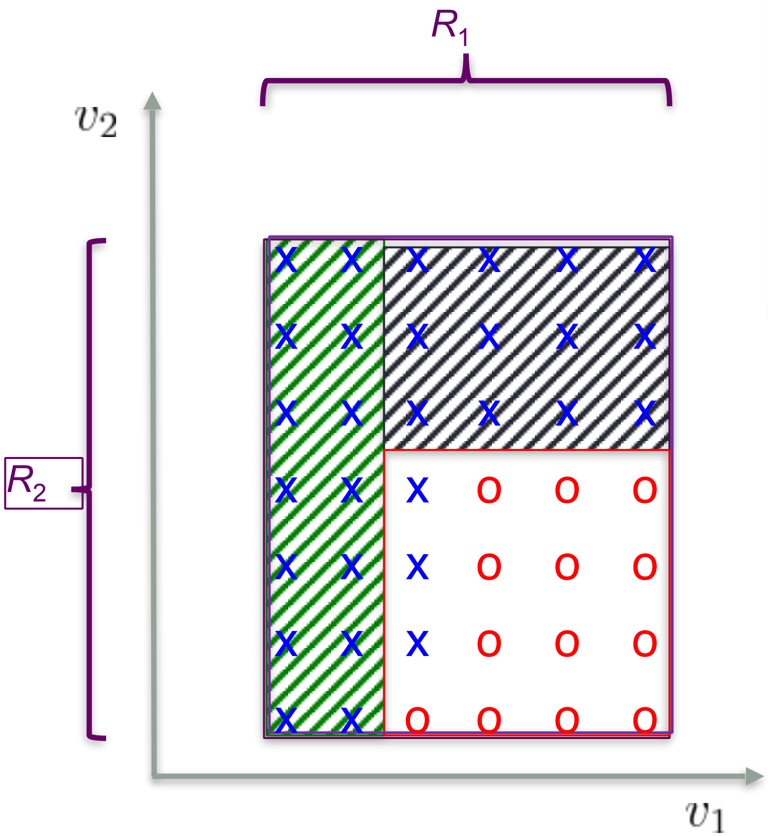

$$\mathcal{C}_1^{\mathcal{U},R} = \overline{\pi_1(\mathcal{U})}^R \times R_2 \times R_3 \times \ldots \times R_k$$

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$$\ldots$$

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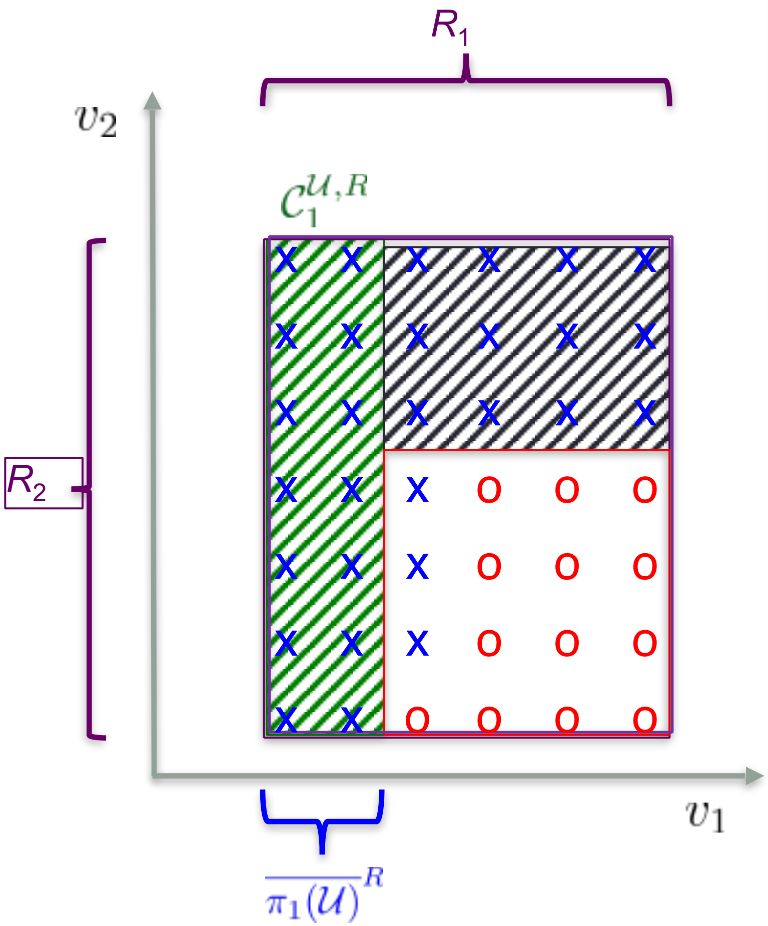

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\end{aligned}
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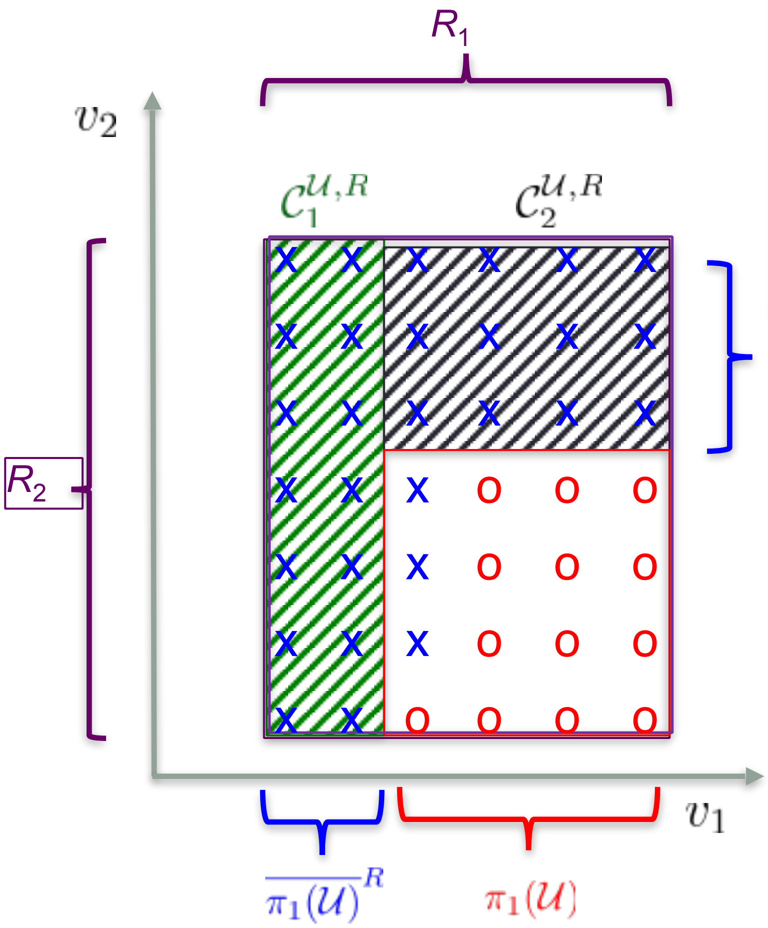

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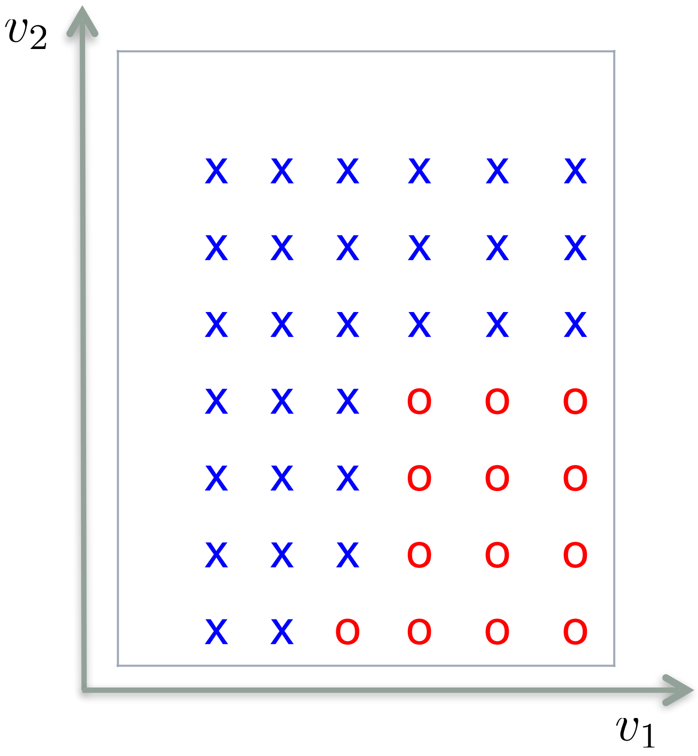
Obs#2 – Lemma 3 and its Corollary (Precondition for Filtering)



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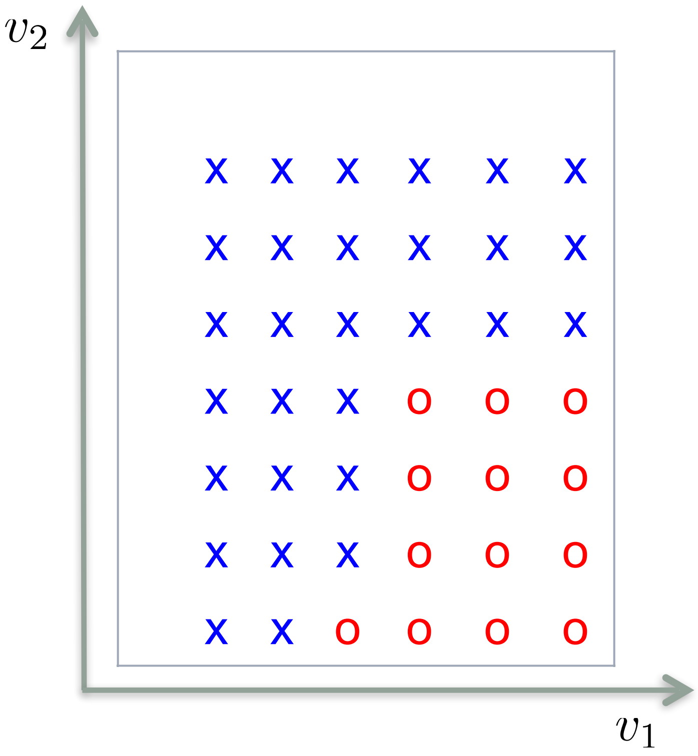

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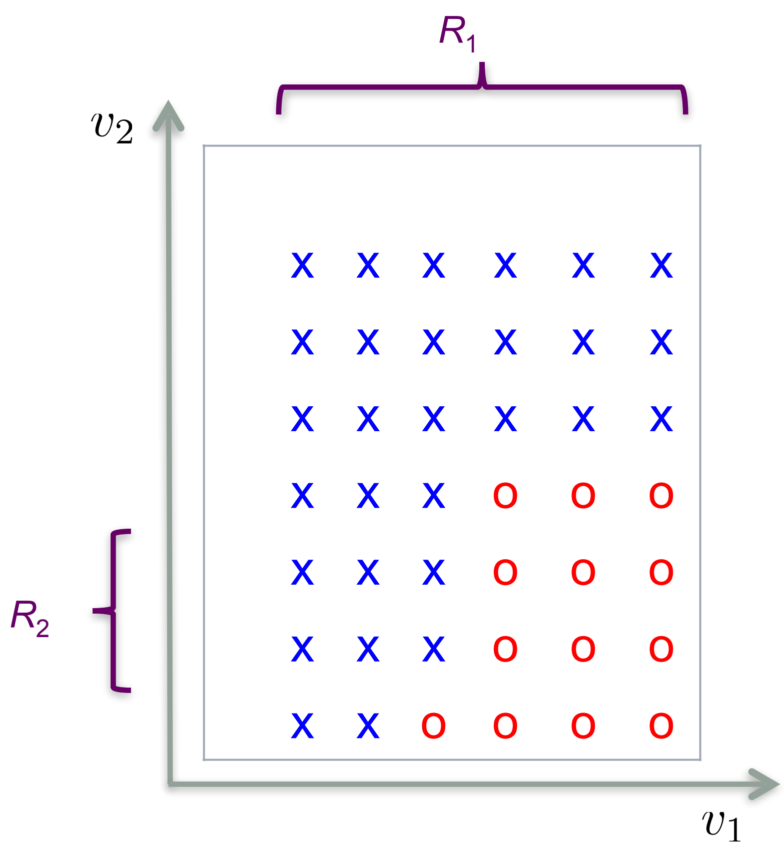

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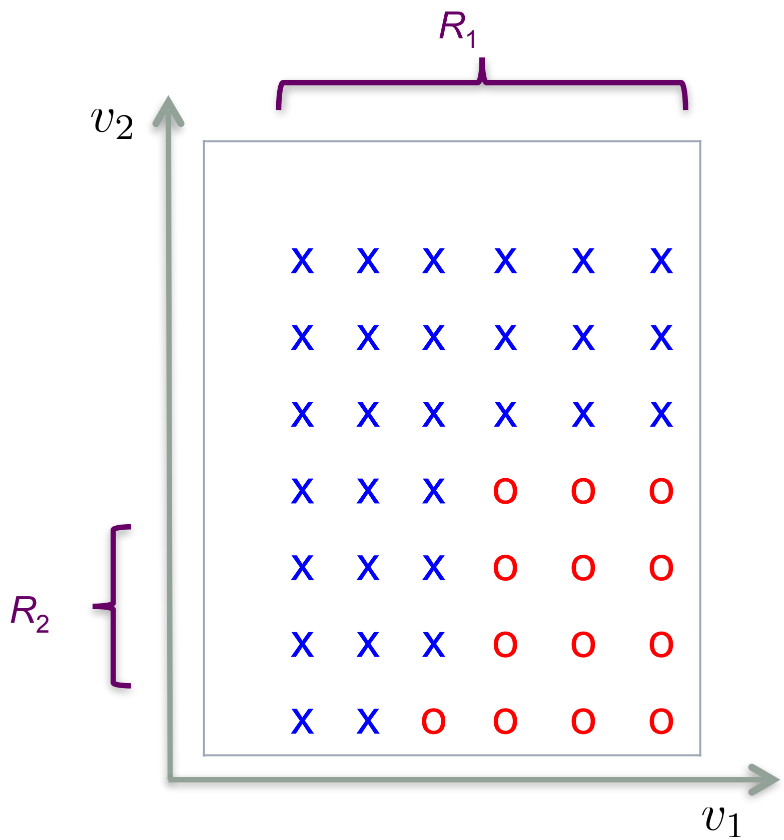

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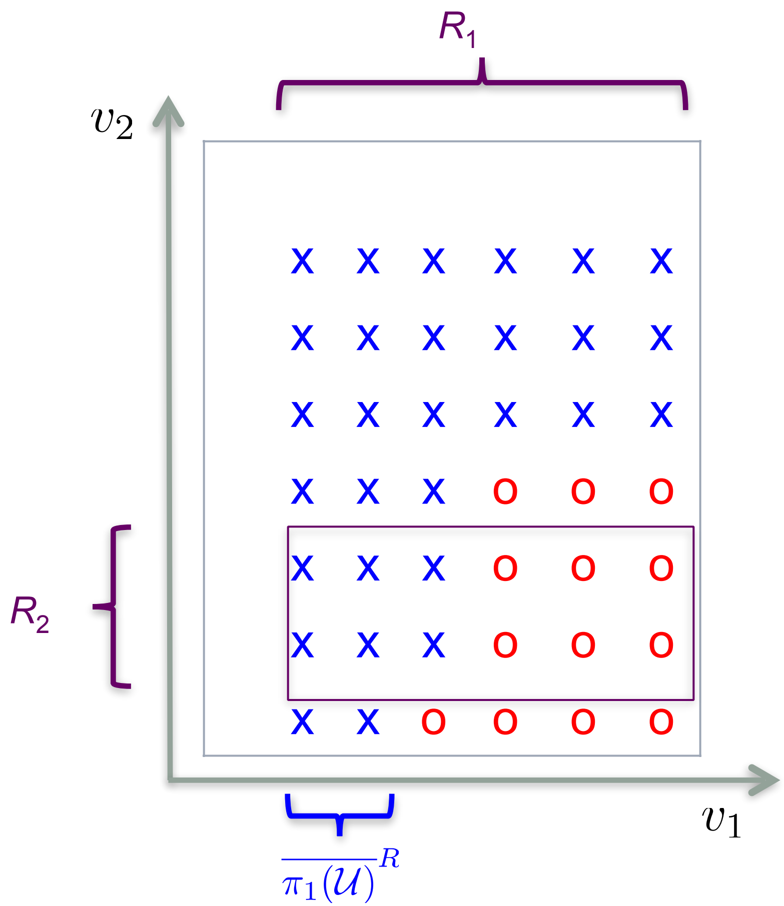

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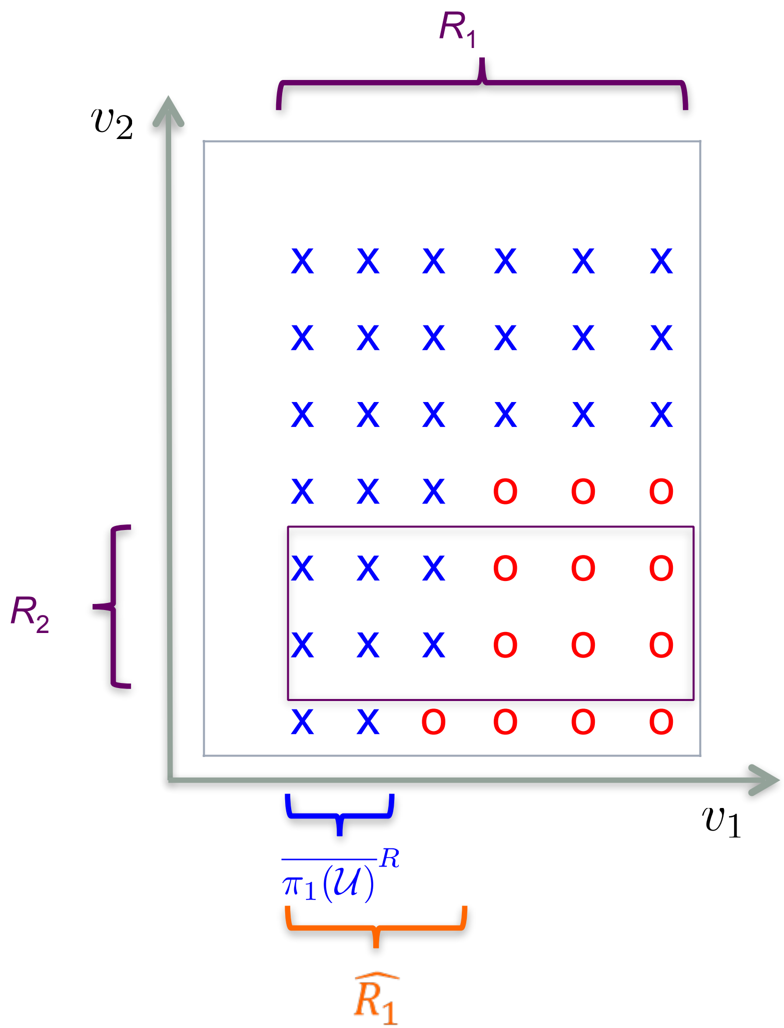

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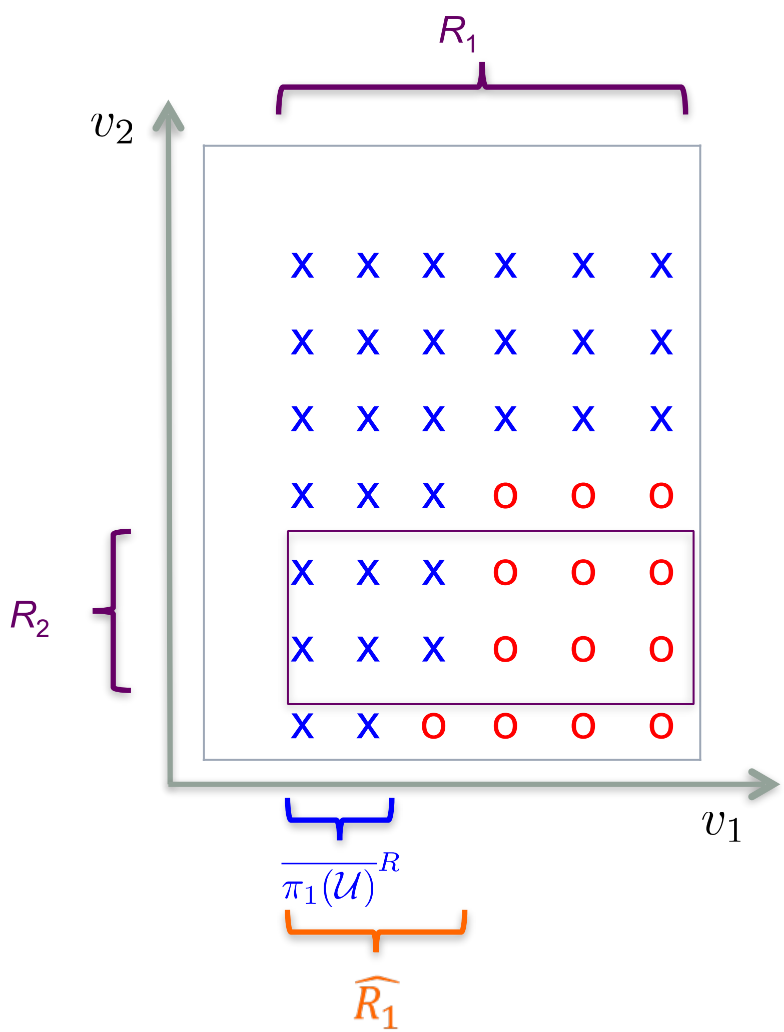

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- Lemma 3: If $\overline{\pi_j(\mathcal{U})}^R$ is non-empty no restriction of any other column is possible
  - Here, $R_1$is restricted to $\widehat{R_1}$
- Corollary: If $\overline{\pi_j(\mathcal{U})}^R$ is non-empty for more than one *j*, no restriction via $\mathcal{U}$ is possible at all

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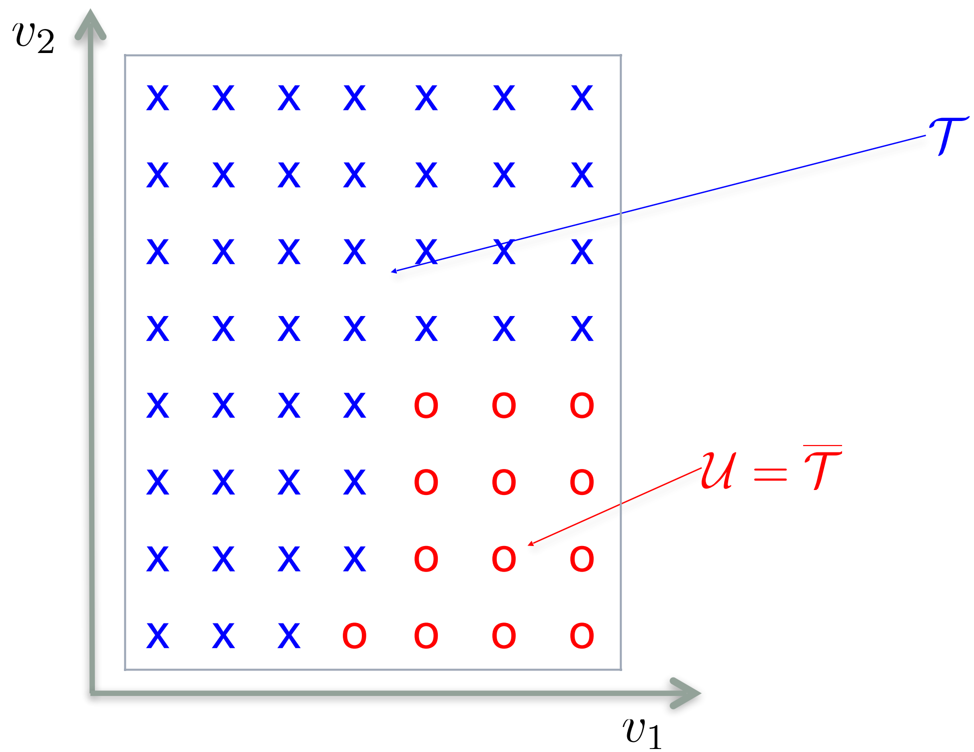
- Example of t-shirt with extended value domains (denoted here by $\mathcal{T}$ instead of $\mathcal{T}^*$)
- Treat $\mathcal{U} := \overline{\mathcal{T}} := \pi(\mathcal{T}) \setminus \mathcal{T}$ as an NVTAB



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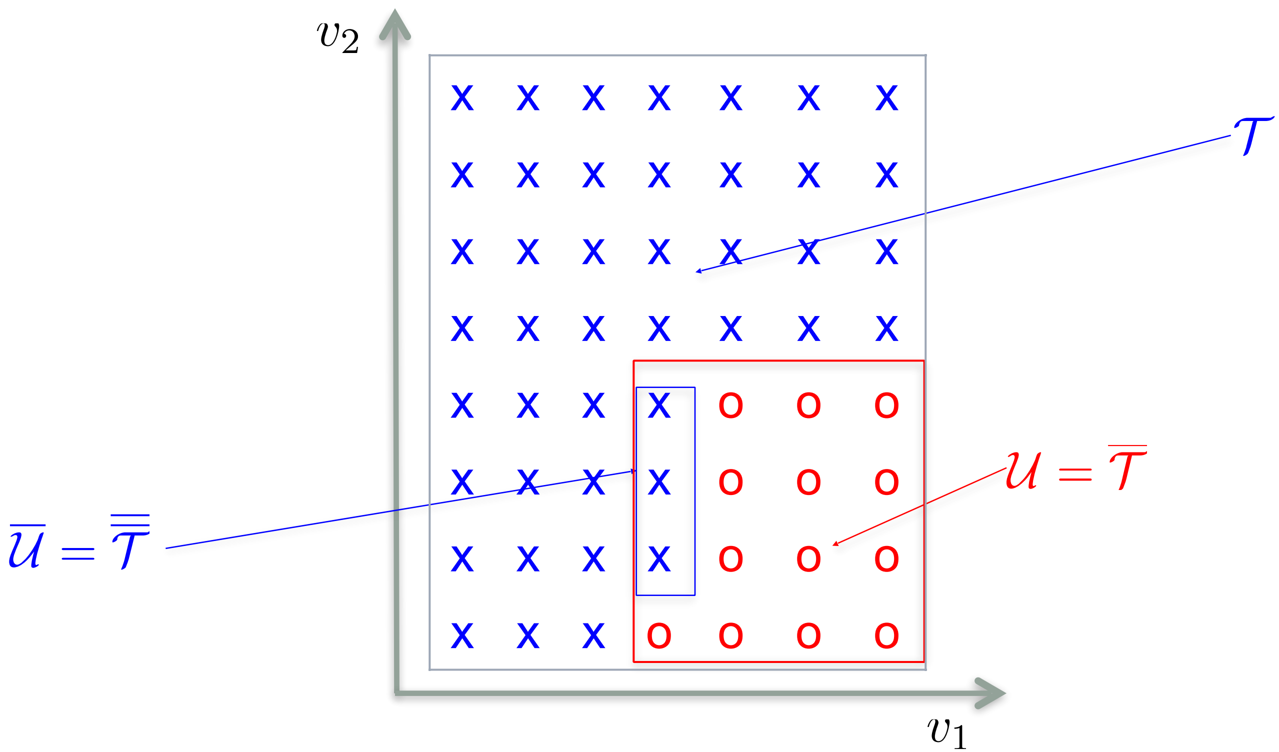

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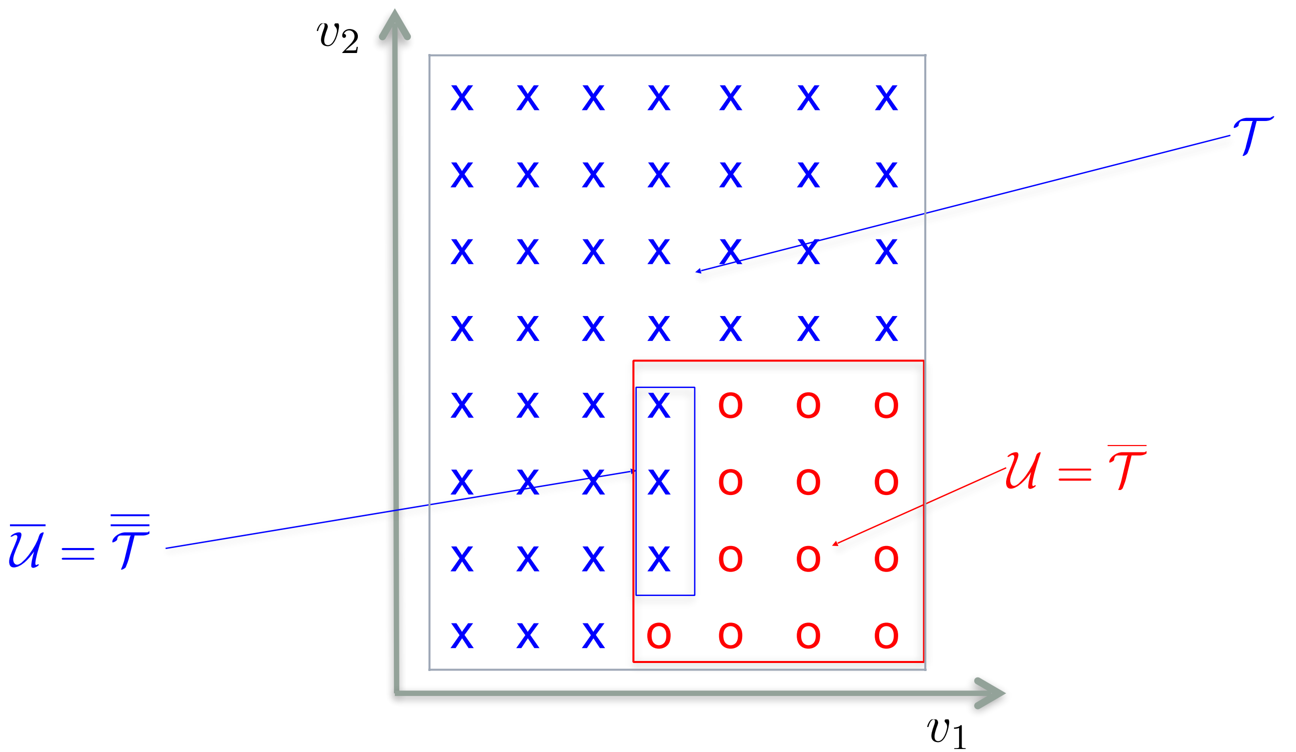
- <u>One way of attempting compression of a VTAB</u>



Obs#3 – Double Negation

- Example of t-shirt with extended value domains (denoted here by $\mathcal{T}$ instead of $\mathcal{T}^*$)
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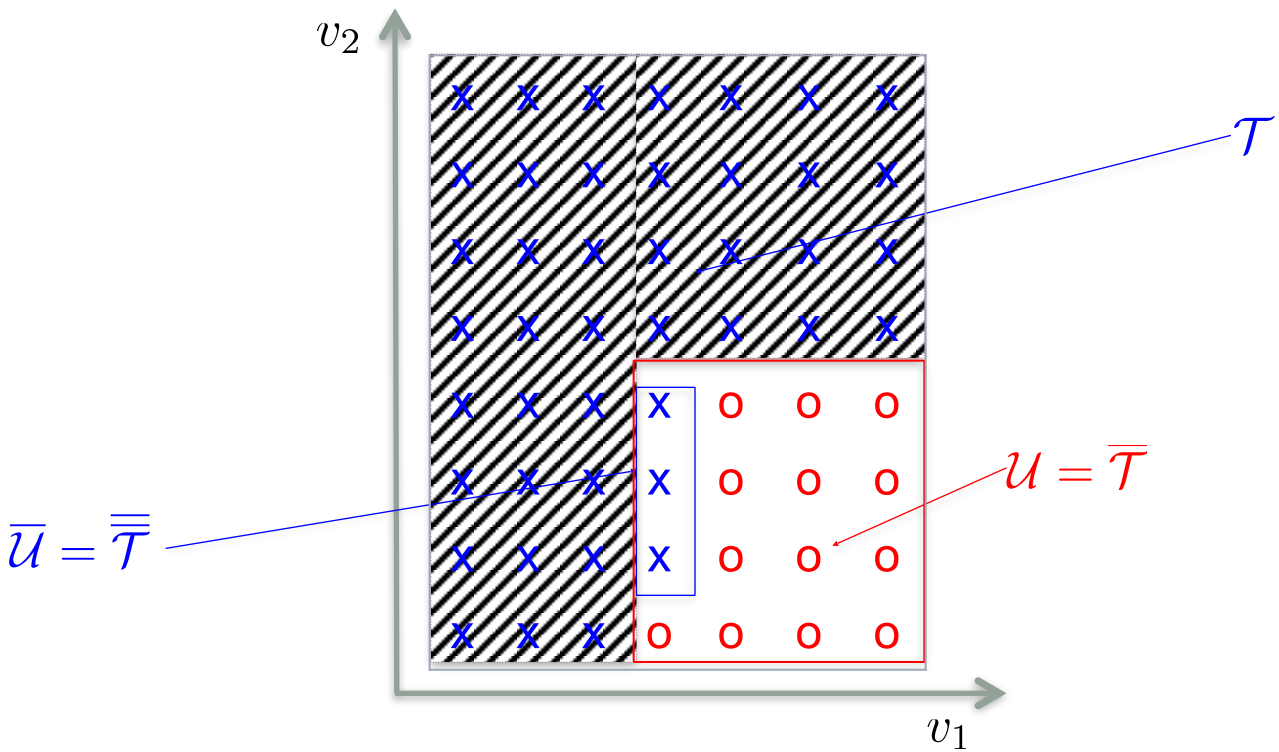

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- Potential performance advantages:
  - Distinction run-time/maintenance-time: smaller $\overline{\mathcal{U}}$
  - Simple test due to Lemma 3/Corollary
  - Double negation one form of potentially compressing tables

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- Advantage likely in the absence of general compression

Thank You