Effective theories of gravity

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3 scales of finite temperature field theory:

$$\int_0^{1/T} d\tau \int d^3x \, \frac{1}{4} F_{\mu\nu}^2(\tau, \mathbf{x})$$

πT

$$\int d^3x \left[\frac{1}{4} F_{ij}^2(\mathbf{x}) + \tfrac{1}{2} g^2 T^2 A_0^2 + ...\right]$$

$g T$

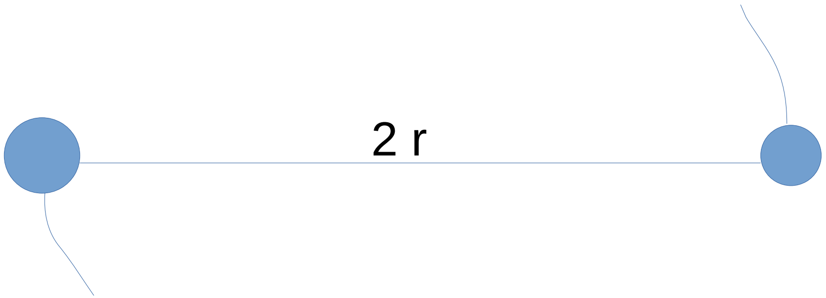
$$\int d^3x \tfrac{1}{4} F_{ij}^2(\mathbf{x})$$

$$\frac{g^2 T}{\pi}$$

K cm
GeV fm

Binary BH gravity also has 3 scales:

$$\frac{c}{v} r = \frac{\lambda_{\rm rad}}{2\pi} \qquad T_H = \frac{\hbar c}{4\pi r_s}$$



$$r_s = \frac{2GM}{c^2} \Rightarrow 2GM \qquad T \sim V : Mv^2 \sim \frac{GM^2}{r} \qquad v^2 = \frac{GM}{4r} = \frac{r_s}{8r}$$

$$J = M\,v\,r = M\sqrt{r\,r_s} \gg \hbar = 1$$

$r_s < r < r/v$

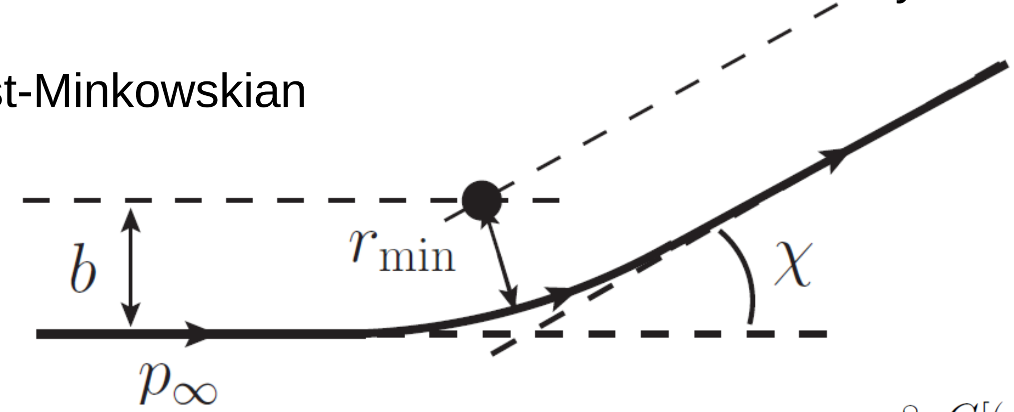
Post-Newtonian = expansion in v^2/c^2, r_s/r

$$\mathrm{NPN} = \mathrm{N} + \mathcal{O}(1/c^{2N})$$

Integrate out systematically r_s

But you can as well do scattering, v → 1 , unbound binary:

Expansion in G, Post-Minkowskian



De Witt 1967 Quantum physics:

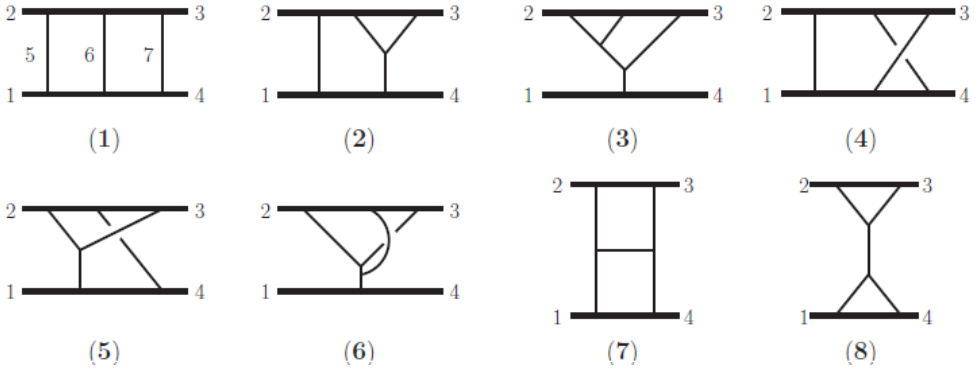
$$M_{\text{Born}}(p_1 + p_2 \to p_3 + p_4) = \frac{8\pi G[(s - 2m^2)^2 - 2m^4]}{-t}$$

1908.01493

Now; two classical particles at

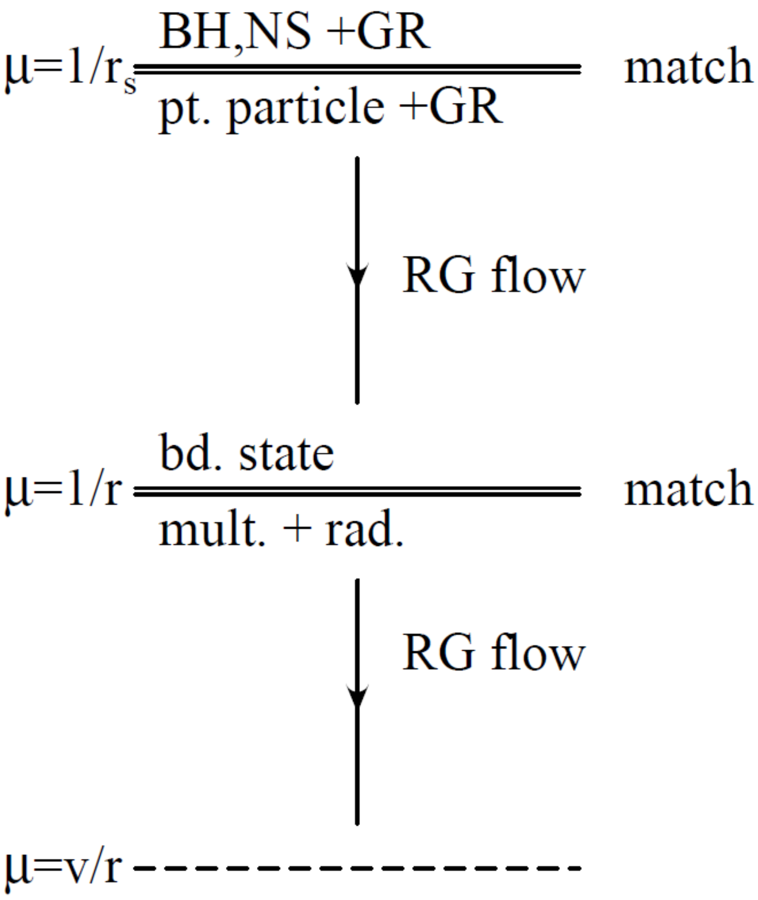
$$\mathbf{x}_1(t),\ \mathbf{x}_2(t)$$

bound or unbound, gravity in dimensionally reduced field theory, 4d to 3d



Goldberger-Rothstein hep-th/0409156
Goldberger, Les Houches 0701129

Foffa-Sturani 1309.3474
Porto 1601.04914



"Integrating out r_s", Point Particle

Develop general relativity so that BH = pointlike particle, pp

For comparison: charged particle in EM field:

$$S = -m\int_{\tau_1}^{\tau_2} d\tau - e\int_{\text{path}} dx^\mu A_\mu(x) + \int d^4x \tfrac{1}{4} F^2 + J^\mu A_\mu$$

Lorentz:
$$\frac{dp^\mu}{d\tau} = e\frac{dx_\alpha}{d\tau}F^{\alpha\mu}$$

=0 =0

$$S[g_{\mu\nu}, x(\tau), \dot{x}(\tau)] = \int d\tau \left[-m + c_R R + c_V R_{\mu\nu}\dot{x}^\mu \dot{x}^\nu + c_E E_{\mu\nu}E^{\mu\nu} + \ldots\right]$$

$$E_{\mu\nu} = R_{\mu\alpha\nu\beta}\dot{x}^\alpha \dot{x}^\beta$$

Spin $S_{ab}\,\omega_\mu^{ab}\dot{x}^\mu$ would enter here!

$$g_{\mu\nu} = \eta_{ab}e^a_\mu e^b_\nu$$

gravrad here!

$$S_{\rm EH}(g_{\mu\nu}) = \frac{1}{16\pi G}\int d^4x\sqrt{g}R(g_{\mu\nu}) \qquad g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}h_{\mu\nu}$$

r_s =2GM

$$\exp\left[iS_{eff}(x_a)\right] = \int Dh_{\mu\nu}(x)\exp\left[iS_{EH}(h) + iS_{pp}(h,x_a)\right]$$

Pointlike BHs at x_α

$$S_{\rm pp} = -m\int dt\sqrt{1+\sqrt{G}h_{\mu\nu}\dot{x}_a^\mu\dot{x}_a^\nu}$$

r

Integrate out gravitons in potential zone k ~ 1/r

$$e^{iS_{\rm eff}(x_a)} = \int \mathcal{D}\bar{h}_{\mu\nu}\underbrace{\mathcal{D}H_{\mu\nu}(t,\mathbf{k})e^{iS_{\rm EH}(\bar{h}+H)+iS_{\rm pp}(\bar{h}+H,x_a)}}_{e^{iS_{\rm eff}(\bar{h},x_a)}}$$

Action for pps at x_a and radiation field $\bar{h}$

Sample comp of

$$\exp\left[iS_{eff}(x_a)\right] = \int Dh_{\mu\nu}(x)\exp\left[iS_{EH}(h) + iS_{pp}(h, x_a)\right]$$

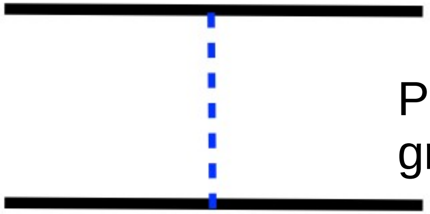
$$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}h_{\mu\nu}$$

$$S_{\rm pp} = -m\int dt\sqrt{1 + h_{00} + ..} = -m\int dt\left(1 + \tfrac{1}{2}\sqrt{G}h_{00}..\right) = ... + \int d^4x J(x)h_{00}(x)$$

$$J(x) = -\tfrac{1}{2}m\sqrt{G}\,\delta^3(\mathbf{x} - \mathbf{x}(t))$$

$$-i\log\int \mathcal{D}h_{00}\exp\left[i\int d^4x(h_{00}\Box h_{00} + J\cdot h_{00})\right] \sim$$

$$\frac{1}{2}\int d^4x_1 d^4x_2 J(x)D_F(x_1 - x_2)J(x_2)] \sim 2\pi m_1 m_2 G\int dt_1 dt_2 D_F(x_1 - x_2)$$



Potential gravitons!

$$k_0 \sim \frac{v}{x} \ll k \sim \frac{1}{x}, \Rightarrow \int \frac{d^4k}{(2\pi)^4}e^{ik\cdot x}\frac{1}{\mathbf{k}^2} = \delta(t)\frac{1}{4\pi|\mathbf{x}|}$$

$$S_{\rm pp} = \int dt\left[\tfrac{1}{2}m_1\mathbf{v}_1^2 + \tfrac{1}{2}m_2\mathbf{v}_2^2 - \tfrac{Gm_1m_2}{|\mathbf{x}_1 - \mathbf{x}_2|}\right]$$

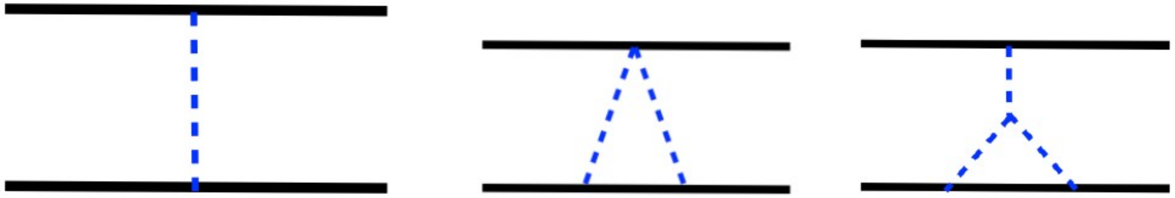
Computing corrections:

$$\frac{1}{-k^2} = \frac{1}{\mathbf{k}^2} + \frac{k_0^2}{\mathbf{k}^4} + .. = \frac{1}{\mathbf{k}^2} - \frac{\partial_t^2}{\mathbf{k}^4} + ..$$

$$\int dt_1 dt_2 \int \frac{dk_0 d^3k}{(2\pi)^4} \frac{\partial_t^2}{\mathbf{k}^4} e^{ik_0(t_1-t_2)+i\mathbf{k}\cdot(\mathbf{x}_1(t_1)-\mathbf{x}_2(t_2))}$$

Time derivative hits $x_a(t)$ and one gets velocities:

$$\int dt \frac{Gm_1m_2}{|\mathbf{x}_1 - \mathbf{x}_2|}\left(\mathbf{v}_1 \cdot \mathbf{v}_2 + \frac{\mathbf{v}_1 \cdot \mathbf{x}_{12}\ \mathbf{v}_2 \cdot \mathbf{x}_{12}}{|\mathbf{x}_1 - \mathbf{x}_2|^2}\right)$$



Gives you

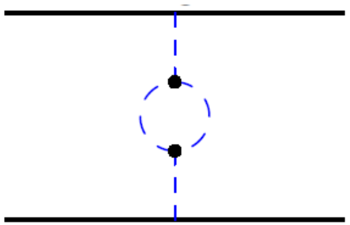
$$L_{\mathrm{EIH}} = \frac{1}{8}\sum_{a=1,2} m_a \boldsymbol{v}_a^4 + \frac{G m_1 m_2}{2r}\left[3(\boldsymbol{v}_1^2+\boldsymbol{v}_2^2) - 7(\boldsymbol{v}_1\cdot\boldsymbol{v}_2) - \frac{(\boldsymbol{v}_1\cdot\boldsymbol{r})(\boldsymbol{v}_2\cdot\boldsymbol{r})}{r^2}\right] - \frac{G_N^2 m_1 m_2 (m_1+m_2)}{2r^2}$$

Einstein-Infeld-Hoffman 1938

$$\frac{GMm}{r}\left(1+\frac{GM}{r}\right)$$

1PN

Note: No quantum loops!



$$\frac{\hbar}{mvr} = \frac{\hbar}{J} <<< 1$$

From L you get EoM (see textbooks)

and no ghosts!

no

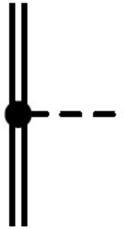
(mass renorm)

More generally, choose a KK type metric (Kol-Smolkin 0712.4116)

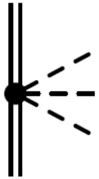
$$g_{\mu\nu} = e^{2\phi/\Lambda}\begin{pmatrix} -1 & A_j/\Lambda \\ A_i/\Lambda & e^{-c_d\phi/\Lambda}\gamma_{ij} - A_iA_j/\Lambda^2 \end{pmatrix} \qquad \gamma_{ij} \equiv \delta_{ij} + \sigma_{ij}/\Lambda, \quad h_{\mu\nu} = (\phi,\ A_i,\ \sigma_{ij})$$

$$c_d = 2\frac{(d-1)}{(d-2)}$$

$$S_{pp} = -m\int d\tau = -m\int dt\ e^{\phi/\Lambda}\sqrt{\left(1-\frac{\vec{A}\cdot\vec{v}}{\Lambda}\right)^2 - e^{-c_d\phi/\Lambda}\left(v^2+\frac{\sigma_{ij}}{\Lambda}v^iv^j\right)}$$



$$= -\frac{im}{m_{Pl}}\int dt\int_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}}\frac{1+\mathbf{v}^2}{\sqrt{1-\mathbf{v}^2}}$$

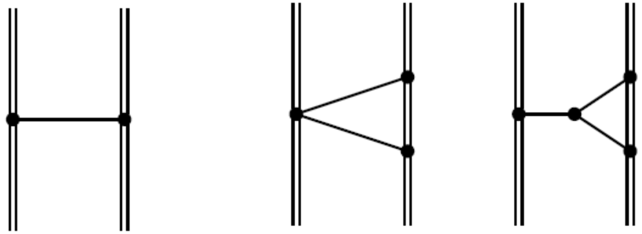


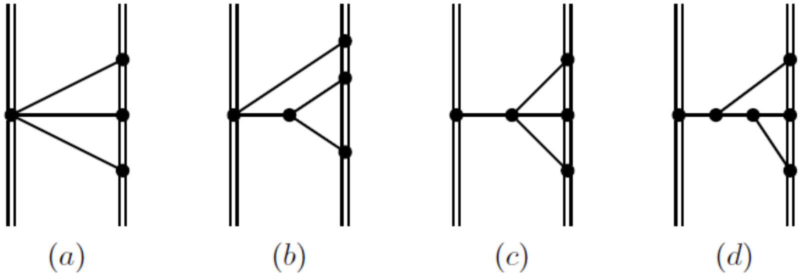
$$= -\frac{im}{m_{Pl}^3}\int dt\int_{\mathbf{k},\mathbf{q},\mathbf{p}} e^{i(\mathbf{k}+\mathbf{q}+\mathbf{p})\cdot\mathbf{x}}\frac{1+11\mathbf{v}^2+11\mathbf{v}^4+\mathbf{v}^6}{(1-\mathbf{v}^2)^{5/2}}$$

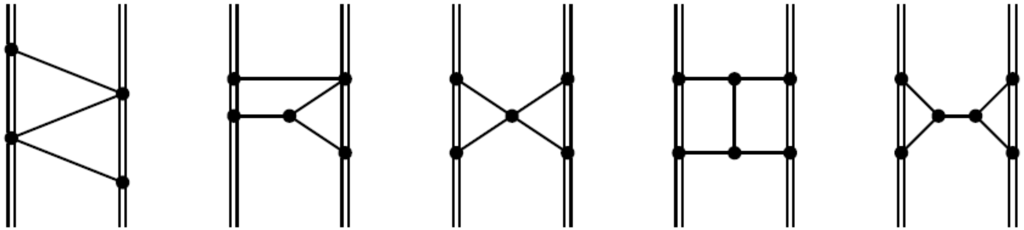
$$\langle T\sigma^{ij}_{\mathbf{p}}(t_a)\sigma^{kl}_{\mathbf{q}}(t_b)\rangle = -(2\pi)^3\delta^3(\mathbf{p}+\mathbf{q})\frac{iP^{ij,kl}}{\mathbf{p}^2}\delta(t_a-t_b)$$

Dimensional reduction 4 to 3!

2PN Gilmore-Ross 0810.1328







$$L_{2PN} = \frac{m_1 \mathbf{v}_1^6}{16} + \frac{G m_1 m_2}{r}\left(\frac{7}{8}\mathbf{v}_1^4 - \frac{5}{4}\mathbf{v}_1^2 \mathbf{v}_1 \cdot \mathbf{v}_2 \cdot \right.$$

$$+ G m_1 m_2 \left(\frac{1}{8}\mathbf{a}_1 \cdot \mathbf{n} \mathbf{v}_2^2 + \frac{3}{2}\mathbf{a}_1 \cdot \mathbf{v}_1 \mathbf{n} \cdot \mathbf{v}_2\right.$$

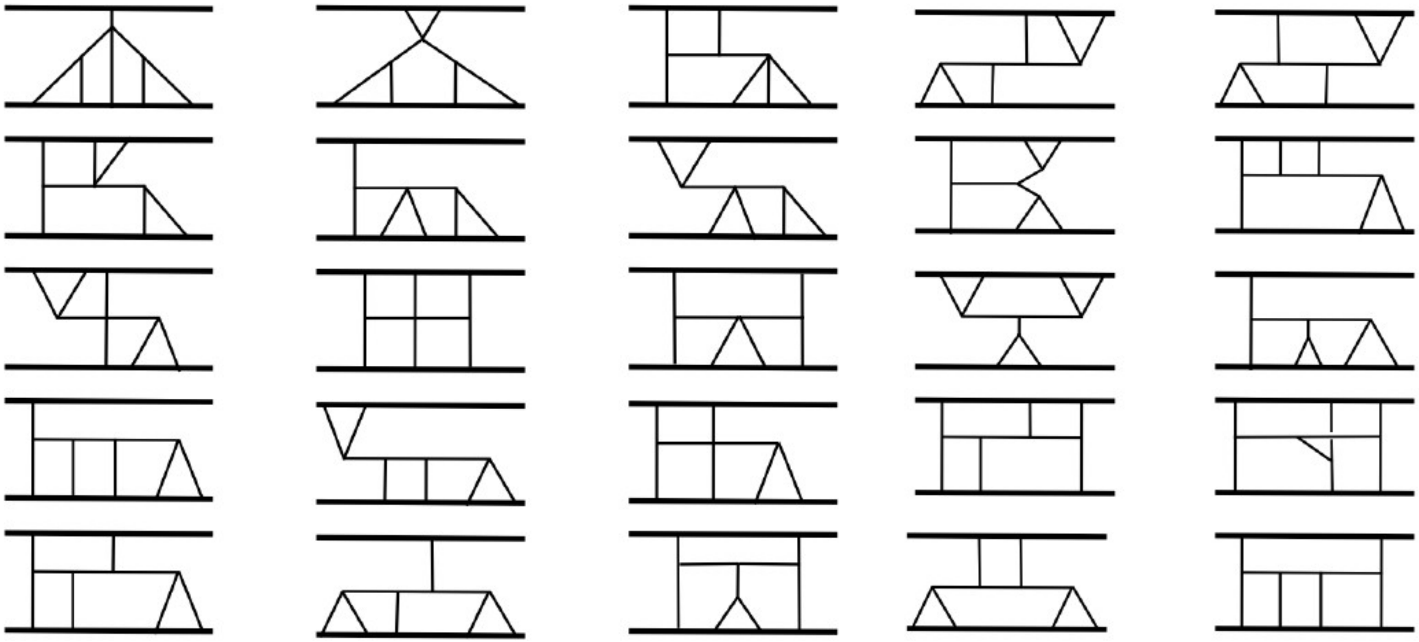
$$+ G m_1 m_2 r \left(\frac{15}{16}\mathbf{a}_1 \cdot \mathbf{a}_2 - \frac{1}{16}\mathbf{a}_1 \cdot \mathbf{n} \mathbf{a}_2 \cdot \mathbf{n}\right) + \frac{G^3 m_1 m_2^3}{2r^3} + \frac{3G^3 m_1^2 m_2^2}{2r^3} + (1 \leftrightarrow 2); + \ldots\ldots$$

$$h_{\mu\nu} = (\phi,\ A_i,\ \sigma_{ij})$$

You get even accelerations!
Can be eliminated using lower order EoM (change of gauge)

4PN topologies: to be filled in with scalar, vector,tensor gravitons

$$h_{\mu\nu} = (\phi,\ A_i,\ \sigma_{ij})$$



Forefront: 5PN, 6PN,…. 6PM, 7PM,...

From a talk by Blümlein on 5PN;

Generation of 962719 Feynman diagrams QGRAF [Nogueira 1993]; performing the Lorentz algebra and further steps FORM 3.0 [Vermaseren 2001-]; Reduction to master intergrals Crusher [Marquard, Seidel].

188533 diagrams are finally contributing.

(I thank York Schröder, Biobio Univ, for pointing out Blümlein’s talk)

Last piece of entertainment from 2008.09389 G^6 PM

TABLE I: Numerical values of the Q_{nk} integrals with 200-digit accuracy.

Q_{20}	524.76729218021258434273595570310175847614199955736901193772871123849883983009771209390703715819606083170623899520567705206794678374496647513473011101045588318417017082934721207112410611316586134856 79

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = 0$$

Using an integer relation algorithm PSLQ one finds that this is

$$\frac{25883}{1800} + \frac{22333}{140}\mathrm{K} - \frac{625463}{3360}\pi - \frac{361911}{560}\pi \ln 2 + \frac{99837}{160}\pi\zeta(3)$$

Katalan’s const

	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN		
1PM	$(1$	$+\ v^{2}$	$+\ v^{4}$	$+\ v^{6}$	$+\ v^{8}$	$+\ v^{10}$	$+\ v^{12}$	$+\ v^{14}$	$+\ \ldots)$	G^{1}
2PM		$(1$	$+\ v^{2}$	$+\ v^{4}$	$+\ v^{6}$	$+\ v^{8}$	$+\ v^{10}$	$+\ v^{12}$	$+\ \ldots)$	G^{2}
3PM			$(1$	$+\ v^{2}$	$+\ v^{4}$	$+\ v^{6}$	$+\ v^{8}$	$+\ v^{10}$	$+\ \ldots)$	G^{3}
4PM				$(1$	$+\ v^{2}$	$+\ v^{4}$	$+\ v^{6}$	$+\ v^{8}$	$+\ \ldots)$	G^{4}
5PM					$(1$	$+\ v^{2}$	$+\ v^{4}$	$+\ v^{6}$	$+\ \ldots)$	G^{5}
6PM						$(1$	$+\ v^{2}$	$+\ v^{4}$	$+\ \ldots)$	G^{6}
							$\vdots$			