1. There are two consumers A and B both of whom get utility from private consumption c_i, $i \in \{A, B\}$ and a public good g. Both consumers have wealth 100. Unit prices of private and public goods are $p = p_g = 1$. The agents have utility functions

$$U_A(c_A, g) = c_A + g$$

and

$$U_B(c_B, g) = c_B + \sqrt{g}$$

The agents make independent and simultanous decisions regarding the supply/purchase of the public good such that $g = g_A + g_B$. So this is a non-cooperative game. Determine its equilibrium. Is there a natural way to improve the situation?

2. Consider a duopoly with Cournot competition. The inverse demand is given by $p = 1 - q$ where q is the total amount of the good in the market. Production costs are given by $c_i(q_i) = \gamma_i q_i$, $i \in \{1, 2\}$ where γ_i are positive constants. It is common knowledge that $\gamma_1 = 0.2$. Firm 2 has with equal probabilities $\gamma_2 = 0.3$ or $\gamma_2 = 0.1$ but this is firm 2's private information. Everything else in the situation is common knowledge. Determine the Bayesian equilibrium of the game.

3. Consider the following normal form game

	α	β	γ	δ	ϵ	ζ	η	θ
a	$3,2$	$-2,4$	$2,6$	$5,4$	$5,4$	$0,0$	$3,7$	$7,7$
b	$3,3$	$0,5$	$1,1$	$4,4$	$4,4$	$1,8$	$10,9$	$2,5$
c	$4,1$	$0,0$	$5,-1$	$3,0$	$3,0$	$1,-1$	$5,0$	$0,0$
d	$3,0$	$1,4$	$2,3$	$3,3$	$3,2$	$0,1$	$2,3$	$3,3$
e	$0,0$	$0,3$	$3,6$	$6,6$	$6,5$	$0,0$	$5,6$	$8,10$
f	$1,2$	$0,2$	$2,2$	$4,1$	$8,3$	$1,2$	$1,1$	$5,2$
g	$2,4$	$0,2$	$3,3$	$5,5$	$6,6$	$2,7$	$4,5$	$6,6$
h	$2,1$	$0,1$	$16,9$	$0,9$	$5,5$	$1,3$	$7,7$	$9,7$

Find two mixed strategy equilibria such that no same strategy is used in them.

4. A seller values an object at zero, and has it for sale. A potential buyer has private valuation that is determined by a draw from a uniform distribution on $[0, 1]$. The seller makes a take-it-or-leave-it offer to the buyer.

i) Determine the offer that maximises expected revenue.

ii) If there are two identical buyers whose valuations are independent draws from the uniform distribution, what is the revenue maximising offer?

5. Two players negotiate about a division of a cake by making offers in alternate turns. If an offer is accepted the cake is divided according to the offer. Otherwise the player who rejected the offer makes a new one. Making an offer

and responding to it takes one period. There are altogether five periods after which the cake vanishes if no offer has been accepted. Players discount utility by factor $0 < \delta < 1$.

i) Determine the subgame perfect equilibrium.

ii) Determine a Nash equilibrium that is not subgame perfect.