Exercise set 3.

1. A risk neutral agent is hired to perform a task. Effort is given by $e \in [0,1)$, and cost of effort by a convex function c, such that $lim_{e\to 1} c(e) = \infty$ and $c(0) = 0$. Outcome can be either failure, $q = 0$, or success, $q = 1$, and effort level e results in success with probability e. The principal is risk averse with utility function $U(q - w)$ where w is the wage paid to the agent. The agent's outside option is worth zero.

i) Assume that e is observable and determine the first-best contract. Interprete it.

ii) Assume that e is not observable and solve for the second best contract.

2. Assume that prisoners' dilemma game is played repeatedly ad infinitum. Construct an asymmetric pair of strategies that constitute a Nash equilibrium for sufficiently high discount factors.

3. i) Players play first the battle of sexes game and then prisoners' dilemma without discounting. Determine the Nash equilibria of the game.

ii) Players play first the prisoners' dilemma and then the battle of sexes game. Determine the Nash equilibria of the game.

Let the games be given by

	c	d
c	$2,2$	$0,3$
d	$3,0$	$1,1$

and

	bo	ba
bo	$3,2$	$0,0$
ba	$0,0$	$2,3$

4. A risk neutral investor has 1000000 to invest in an entrepreneur's project. The entrepreneur's utility for wealth w and effort e is given by $u(w,e) = log(1+w) - Ae$. If the project is successful it is worth 3000000 and if not it is worth zero. Given effort $e = 0$ the project succeeds with probability $\frac{1}{2}$, and given effort $e = 1$ it succeeds with probability $\frac{4}{5}$. What is the smallest value of A such that the investor does not want to induce the entrepreneur to choose effort level $e = 1$.

5. An entrepreneur needs I to finance a project. There are two types of entrepreneurs good and bad, $\{g, b\}$, $0 < b < g < 1$. A bad entrepreneur's project returns R_b with probability b, and a good entrepreneur's project returns R_g with probability g. Assume that $bR_b = gR_g > I$. If an entrepreneur gets the needed financing s/he commits to a debt contract (D, L) where D is the amount of repayment, and $L \leq \bar{L}$ is the amount of illiquid collateral that is lost in case of default. Proportion π of the entrepreneurs are of good type. Determine a

separating equilibrium where the entrepreneurs suggest a debt contract to the financiers.