Games

Classifications of games

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Non-co-operative games and co-operative games

- In non-co-operative games the focus is on individual players and their actions, while in co-operative games one does not consider what individual players do; quite to the contrary the main role is played by a function that tells the worth of each coalition.
- The focus of these lectures is non-co-operative games.

Normal form games and extensive form games

- This is a classification of non-co-operative games.
- In normal form games the players can be thought to choose their action simultaneously, while in extensive form games the sequential structure of the strategic situations is important.
- We start with normal form games.
- Sometimes people make distinction between static games and dynamic games when referring to the normal form games and extensive form games, but it is slightly misleading.

Non-co-operative games

Games with complete and incomplete information

- This classification also pertains to non-co-operative games.
- It makes a huge difference whether there is complete information or whether the players do not know some relevant aspects of their opponents.

Non-co-operative games

- Why is game theory worth studying?
- It constitutes the main technical tool in many social sciences as well as in other sciences like biology.
- Most of the interesting interaction between decision makers, be they individuals or firms or countries, is strategic, i.e., one's optimal behaviour depends on other decision maker's behaviour.
- At this point it is typical to mention some examples but I find it much more challenging to find important situations which are not of strategic nature.
- None come to mind.

Non-co-operative games

- Game theory is based on von Neumann-Morgenstern preferences.
- Examples of normal form games:
- Prisoners' dilemma

	C	D
C	$2,2$	$0,3$
D	$3,0$	$1,1$

Non-co-operative games

Battle of the sexes

	Bo	Ba
Bo	$2, 1$	$0, 0$
Ba	$0, 0$	$1, 2$

Stag hunt

	S	H
S	$2,2$	$0,1$
H	$1,0$	$1,1$

Hawk and dove

$$\begin{array}{ccc} & H & D \\ H & \frac{v-c}{2}, \frac{v-c}{2} & v, 0 \\ D & 0, v & \frac{v}{2}, \frac{v}{2} \end{array} \quad \text{where } c > v.$$

Game with many equilibria

	L	R
U	$1,1$	$0,0$
M	$1,1$	$2,1$
D	$0,0$	$2,1$

Co-ordination game

	S	H
S	$9,9$	$0,0$
H	$0,0$	$1,1$

Non-co-operative games

Which are plausible outcomes? Why?

- One principle is dominance.
- There are two types.
- Strict dominance and weak dominance.
- One should expect that a strictly dominated action is never chosen.
- One could iteratively remove all strictly dominated actions but this does not typically lead to a unique outcome.
- Iteratively removing weakly dominated actions may lead to different outcomes depending on the order of removal.
- The bottom line is that no form of iteratively removing dominated actions/strategies provides a foundation for a solution to games.

- To consider iterative dominance arguments, and to proceed anyway, we first need to carefully formalise what a normal form game is.

Definition

A normal form game is given by $\Gamma = \left\{N, \{A_i\}_{i \in N}, \{u_i\}_{i \in N}\right\}$ where $N = \{1, 2, ..., n\}$ is the set of players, A_i is player i's set of actions, and u_i is player i's utility function $u_i : \Pi_{j \in N} A_j \to R$.
Strict dominance. In a normal form game $\Gamma = \left\{N, \{A_i\}_{i \in N}, \{u_i\}_{i \in N}\right\}$ an action $a_i \in A_i$ is strictly dominated if there is a different action $a_i' \in A_i$ such that

$$u_i(a_i', a_{-i}) > u_i(a_i, a_{-i})$$

for all $a_{-i} \in A_{-i}$.

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$$u_i(a_i', a_{-i}) \geq u_i(a_i, a_{-i})$$

for all $a_{-i} \in A_{-i}$, and

$$u_i(a_i', a_{-i}) > u_i(a_i, a_{-i})$$

for some $a_{-i} \in A_{-i}$.

Non-co-operative games

- The solution concept that is adopted is that of Nash-equilibrium.

Definition

In a normal form game $\Gamma = \left\{N, \{A_i\}_{i \in N}, \{u_i\}_{i \in N}\right\}$ an n-tuple of actions $(a_1, ..., a_n)$ is a Nash-equilibrium if

$$u_i(a_i, a_{-i}) \geq u_i(a_i', a_{-i})$$

for all $a_i' \in A_i$, and for all $i \in N$.

- One of the advantages of Nash-equilibrium is that it usually exists in situations of interest.
- One of the disadvantages is that there are typically multiplicity of them.

- A curious example about strict dominance.
- Let the set of players be $N = \{1,2\}$,the action sets $A_1 = A_2 = [0,1]$,and the utility functions $u_i : A_i \times A_j \to R$

$$\begin{aligned} u_i(x,y) &= x \text{ if } x < 1 \\ u_i(1,y) &= 0 \text{ if } y < 1 \\ u_i(1,1) &= 1 \end{aligned}$$

- Each action except 1 is strictly dominated, and $(1,1)$ is the unique Nash-equilibrium.

- Eliminating all actions $A_i \setminus \{1, x\}$, $x < 1$, gives the following 2x2 game

	1	x
1	$1,1$	$0,x$
x	$x,0$	x,x

- Even using iterative strict dominance the order of removal of actions affects the set of Nash-equilibria.