Signalling

Lecture 11

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- In signalling models the informed party makes the first move.
- His/her choice may reveal some of his/her private information.
- In the standard model there are good and bad types, and the good types want to signal their goodness.
- To be able to do this the signal they send must be costly.
- Sadly, the signalling models are plagued by a multitude of equilibria.
- In separating equilibria the different types choose different actions.
- In pooling equilibria different types choose the same action.

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- We present the educational model by Spence.
- There is either a pool of employees where some are of low productivity θ_l and some of high productivity θ_h, or one employee who is either of low productivity or of high productivity.
- The utility of a worker of type θ who acquires education level e and gets wage w is given by $u(w) - C(e, \theta)$.
- It is assumed that $u' > 0$, $u'' < 0$, $\frac{\partial C}{\partial e} > 0$, $\frac{\partial^2 C}{\partial e^2} > 0$, $\frac{\partial^2 C}{\partial e \partial \theta} < 0$.
- Notice that education does not affect productivity but it is cheaper for the more productive type.
- The productivity is assumed private information.

- The cross-partial derivate condition is the sorting condition which makes separation of types possible.
- Assume that the market for employees is competitive so that each employee gets his/her expected marginal product.
- Assume that the expectation that a worker with education level e is of type θ_l with probability $\mu(e)$.
- Then s/he is paid $w(e) = \mu(e)\theta_l + (1 - \mu(e))\,\theta_h$.
- The a-priori-beliefs of the employers are given by μ_0.

A perfect Bayesian equilibrium in pure strategies comprises strategies (e_l^*, e_h^*, w^*) and a system of beliefs μ^* such that

1. Given w^* each type i's level of education satisfies $e_i^* \in argmax_e [u(w^*(e) - C(e, \theta_i))]$.
2. The wage function is given by $w^*(e) = \mu^*(e)\theta_l + (1 - \mu^*(e))\theta_h$.
3. The beliefs are consistent with strategies so that if $e_l^* \neq e_h^*$ then $\mu^*(e_l^*) = 1$, and if $e_l^* = e_h^*$ then $\mu^*(e_l^*) = \mu_0$.

For all other education levels one is free to choose the beliefs; this generates the multitude of equilibria.

- Let us study a separating equilibrium.
- In such an equilibrium the types choose education levels $e_l^* < e_h^*$.
- Since education is costly it is clear that $e_l^* = 0$, and a low productivity agent gets wage θ_l.
- In equilibrium it must be the case that a low productivity agent does not choose education level e_h^* or

$$u(\theta_l) - C(0, \theta_l) \geq u(\theta_h) - C(e_h^*, \theta_l)$$

- It is clear that this holds only if e_h^* is high enough.

- Analogously, it must be the case that

$$u(\theta_h) - C(e_h^*, \theta_h) \geq u(\theta_l) - C(0, \theta_h)$$

- It is clear that this holds only if e_h^* is low enough.

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- Let us next study a pooling equilibrium.
- In such each type chooses the same education level e^*.
- This does not convey any information to the employers and each type gets wage $w = \mu_0 \theta_l + (1 - \mu_0) \theta_h$.
- It is clear that now e^*cannot be too high since otherwise low productivity employees would not acquire the education but would be satisfied with wage θ_l.
- Since education is pure waste in this model the best pooling equilibrium is such that every worker acquires zero level of education.

- Assume that $u(x) = x$, $C(e, \theta_l) = 2e$, $C(e, \theta_h) = e$, $\mu_0 = \frac{1}{2}$, $\theta_l = 0$ and $\theta_h = 1$.
- Now, the least-cost separating equilibrium is such that $e_l^* = 0$, $e_h^* = \frac{1}{2}$, and the beliefs when anyone chooses any other level of education are such that with probability one such an employee is regarded as a low productivity type.
- A pooling equilibrium where each type acquires zero level of education is not a credible equilibrium in the following sense.
- It is supported by beliefs by which anyone choosing any other level of education is of low productivity.

- Consider an an employee who chooses e such that $\frac{1}{4} < e < \frac{1}{2}$ in the hope of being regarded as a high productivity type.
- Even if s/he were regarded as a high productivity type a low productivity type would increase his/her pay-off from $\frac{1}{2}$ to unity at a cost that exceeds $\frac{1}{2}$.
- The high productivity type would increase his/her pay-off from $\frac{1}{2}$ to unity at a cost less than $\frac{1}{2}$.
- Thus, the only sensible conclusion of employers is that the deviator is of high productivity.
- The pooling equilibrium does not satisfy so called intuitive criterion.