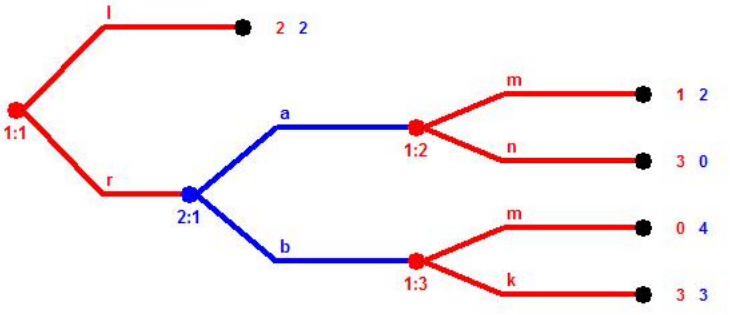
Extensive game with imperfect information

Sixth lecture

November 12, 2015

Normal form representation of extensive form games

Consider the following game



Normal form representation of extensive form games

- Player 1's strategies are given by
- $\{(lmm), (lmk), (lnm), (lnk), (rmm), (rmk), (rnm), (rnk)\}$.
- Player 2's strategies are given by $\{a, b\}$.
- Now one can construct a normal form game from these strategies such that it corresponds to the extensive form game.

	a	b
lmm	2, 2	2, 2
lmk	2, 2	2, 2
lnm	2, 2	2, 2
lnk	2, 2	2, 2
rmm	1, 2	0, 4
rmk	1, 2	3, 3
rnm	3, 0	0, 4
rnk	3, 0	3, 3

- From this one finds equilibria $((r, m, k); b)$ and $((r, n, k); b)$.

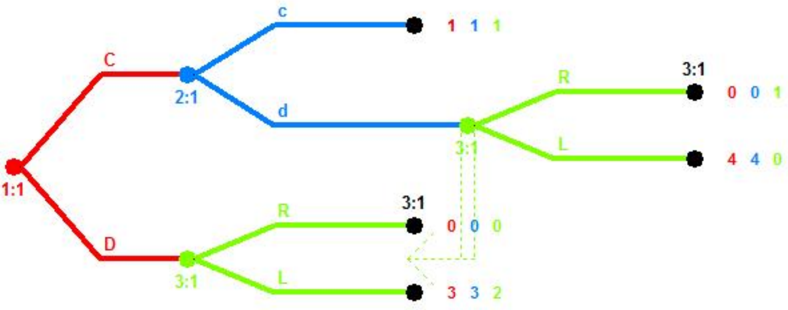
- Instead of going through the general theory we focus on illuminating examples.

- One can think of games with imperfect information such that players do not necessarily know which choices have been made once they make their choices.

- Then one of the major refinements is sequential equilibrium; not all equilibria are such

- An example is a game called Selten's horse.

Selten's horse


c
1 1 1
C
2:1
d
3:1
R
0 0 1
L
4 4 0
3:1
1:1
3:1
R
0 0 0
D
L
3:1
3 3 2

Selten's horse

- In equilibrium we have to say what each player does given his/her BELIEFS.

- It is clear that no pure strategy equilibria exist.

- One equilibrium is such that player 1 chooses D with probability one.

- Player 2 chooses, if s/he ever gets to choose, c with a probability that is between $1/3$ and 1, and player 3 chooses L with probability 1.

- Here player 1's belief is that s/he is in the node following history (D) with probability one.

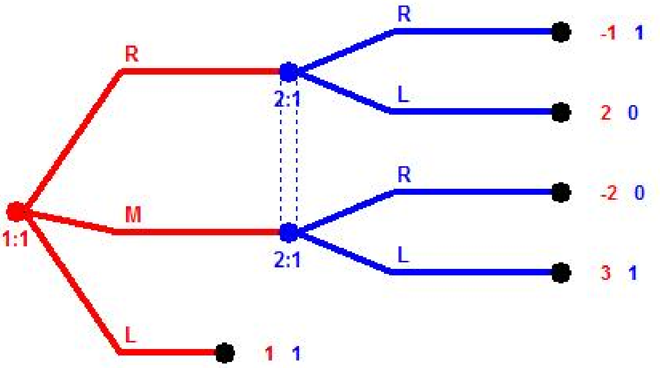
Selten's horse

- This is not a good equilibrium (it is not a sequential equilibrium).
- This is because player 2's choice is not rational.
- S/he should choose d; but then player 1 should choose C, and the whole thing would break down as player 3's beliefs would not be correct.

Selten's horse

- Another type of equilibrium is such that player 1 chooses C with probability one.
- Player 2 chooses c with probability one, and player 3 chooses R with probability between $3/4$ and unity.
- For player 3 to mix to be rational s/he has to have beliefs about which node in his/her information set s/he is at.
- Denote the belief that history has been D by π.
- Choosing L gives then 2π and R $1-\pi$.
- These are equal if $\pi = \frac{1}{3}$.
- There is nothing wrong with this belief; outside the equilibrium path pretty much any beliefs are allowed.

An unknown game


R
-1 1
R
2:1
L
2 0
R
-2 0
M
1:1
2:1
L
3 1
L
1 1

An unknown game

- To solve for equilibria one postulates that player 1's strategy is to choose L, M or R with probability (α, β, γ).
- If $\beta > \gamma$ then player 2 chooses L.
- Thus $(M; L)$ is an equilibrium.
- If $\beta < \gamma$ then player 2 chooses R against which player 1's best response is L, or $\beta = \gamma = 0$ which does not jibe with $\beta < \gamma$.
- If $\beta = \gamma > 0$ then player 2 must choose L and R with probability $\frac{1}{2}$ but then player 1's best response is L.
- If $\beta = \gamma = 0$ then player 2's strategy $(\theta, 1-\theta)$ must satisfy the following conditions:

$$3\theta - 2(1-\theta) \leq 1$$

$$2\theta - (1-\theta) \leq 1$$

- The first condition yields $\theta \leq \frac{3}{5}$ and the second one $\theta \leq \frac{2}{3}$.
- Whenever $0 < \theta \leq \frac{3}{5}$ the tentative equilibrium is supported by player 2's beliefs $(\frac{1}{2}, \frac{1}{2})$.

- Whenever $\theta = 0$ the tentative equilibrium is supported by player 2's beliefs $(p, 1-p)$, for $p \leq \frac{1}{2}$.

- There are two types of 'good' equilibria: $((0,1,0),(1,0))$ and player 2's belief is $(1,0)$, and $((1,0,0),(\theta,1-\theta))$, $\theta \in \left[0, \frac{3}{5}\right]$ and player 2's belief is $\left(\frac{1}{2}, \frac{1}{2}\right)$ if $\theta > 0$ and $(p, 1-p)$, for $p \leq \frac{1}{2}$ if $\theta = 0$.

Observable actions

- Assume that Nature chooses pay-off relevant features of the players (types).
- A player learns his/her own type but nothing about the other players.
- The players can observe all the actions but the Nature's choice.
- In this setting one typically uses as a solution concept perfect Bayesian equilibrium.
- The requirement of the equilibrium concept is that players always optimise againts their beliefs, and that the beliefs are as correct as possible; every time it is possible to update them one uses Bayes's rule.
- In particular, in parts of the game tree which are reached according to the players' strategies the beliefs must be consistent with the strategies.