Moral hazard

- Situations where the principal (who offers a contract) cannot observe the action of the agent (who accepts or rejects the contract).
- The principal can only observe an imperfect signal of the agent's action.
- In a typical situation the principal observes the output but not the effort of the agent.
- The situation is interesting when the agent's and the principal's interests are not aligned.
- The agent has to be 'bribed' to choose the action desired by the principal.

Moral hazard

Example

The agent can choose effort $e \in \{0, 1\}$.
The cost of effort is $c(e) = e$.
Effort $e = 0$ yields output A with probability p and zero with probability $1 - p$.
Effort $e = 1$ yields output A with probability $q > p$ and zero with probability $1 - q$.
The principal offers the agent R if output is A, and $r < R$ if output is zero.
The principal and the agent are risk neutral.

Moral hazard

Example

If the agent chooses $e = 0$ s/he gets $pR + (1 - p)r$.
If the agent chooses $e = 1$ s/he gets $qR + (1 - q)r - 1$.
The latter is greater than the former if $(q - p)(R - r) - 1 \geq 0$.
If the principal wants the agent to choose $e = 1$ s/he chooses a contract that makes the agent indifferent.
Thus, $r = R - \frac{1}{q-p}$.
The principal has to take care that the agent gets at least zero utility.
This happens when $R \geq \frac{1-p}{q-p}$ and the principal chooses $R = \frac{1-p}{q-p}$.

Example

Then the principal gets $q(A-R)-(1-q)r$ which is equivalent to $qA-1$.

Making a take-it-or-leave-it offer for the project to the agent the maximum the principal can get is $qA-1$.

In effect the optimal contract amounts to selling the project to the agent.

This makes plenty of sense since the agent is risk neutral.

When the agent is risk averse the principal must balance efficiency considerations with insurance considerations.

Moral hazard

- Next the standard set-up for moral hazard is given.
- The agent's action set is $\{a_1, ..., a_n\}$.
- The set of possible outcomes is $\{x_1, ..., x_m\}$.
- Action a_i results in outcome x_j with probability $p_{ij} > 0$.
- Only the outcome is observable.
- The only possible contracts are contingent on outcomes; x_j results in pay-off w_j to the agent.

Moral hazard

- The agent's utility is concave (risk averse) in pay-off, and quasilinear in the cost of action, and the principal is risk neutral.
- Action a, outcome x and pay-off w result in utilities $u(w) - a$ for the agent and $x - w$ for the principal.
- Given the contract offer by the principal the agent's problem is

$$max_{a_i} \left(\sum_{j=1}^{m} p_{ij} u(w_j) - a_i \right)$$

- As the principal wants the agent to choose a_i it must be the case that the following incentive constraints hold for $k \neq i$

$$\sum_{j=1}^{m} p_{ij} u(w_j) - a_i \geq \sum_{j=1}^{m} p_{kj} u(w_j) - a_k$$

Moral hazard

- It is assumed that the agent has an outside option (outside the model) which guarantees him/her expected utility $\underline{u}$.
- The contract must guarantee at least this much to the agent giving rise to an IR-constraint

$$\sum_{j=1}^{m} p_{ij} u(w_j) - a_i \geq \underline{u}$$

- Principal aims to choose $(w_1, ..., w_m)$ to maximise his/her expected utility taking into account the agent's behaviour

$$max_{(w_1,...,w_m),a_i} \sum_{j=1}^{m} p_{ij}(x_j - w_j)$$

subject to IC constraints for $k \neq i$ and the IR constraint.

Moral hazard

- The Lagrangian for the problem, assuming a_i is the desired choice by the principal, is given by

$$L = \sum_{j=1}^{m} p_{ij}(x_j - w_j) + \sum_{k \neq i} \lambda_k \left(\sum_{j=1}^{m} p_{ij} u(w_j) - a_i - \sum_{j=1}^{m} p_{kj} u(w_j) + a_k \right) + \mu$$

- The FOC with respect to w_j is given by

$$\frac{1}{u'(w_j)} = \mu + \sum_{k \neq i} \lambda_k \left(1 - \frac{p_{kj}}{p_{ij}} \right)$$

- The next step would be the study of properties of the optimal contract.
- The optimal contract may be, however, highly non-monotonic without strong assumptions about the distribution of outcomes conditioned on the actions.
- Thus, we focus on a two-action two-outcome case.

Moral hazard

- Assume that $x_1 < x_2$ and $0 = a_1 < a_2$.
- Assume that the principal wants the agent to choose a_2.
- Then it must be the case that

$$p_{22}u(w_2) + (1 - p_{22})u(w_1) - a_2 \geq p_{12}u(w_2) + (1 - p_{12})u(w_1)$$

which is equivalent to

$$(p_{22} - p_{12})(u(w_2) - u(w_1)) \geq a_2$$

- It is clear that the closer p_{22} and p_{12} the more powerful incentives, i.e., higher wage the principal must give the agent.

- The IR constraints states

$$p_{22}u(w_2) + (1 - p_{22})u(w_1) - a_2 \geq \underline{u}$$

- If this did not hold as equality the principal could subtract ε from w_1 and w_2 without violating the IC.
- If the IC did not bind the principal could subtract ε from w_2 and add $\frac{p_{22}}{1-p_{22}}\frac{u'(w_2)}{u'(w_1)}\varepsilon$ to w_1.
- For small ε the IR constraint remains the same.
- The expected increase in profits would be $p_{22}\varepsilon - (1 - p_{22})\frac{p_{22}}{1-p_{22}}\frac{u'(w_2)}{u'(w_1)}\varepsilon > 0$ as $u'(w_1) > u'(w_2)$.

Moral hazard

- Now we have two equalities, and a little thinking shows that instead of solving for w_i we can solve equally well for $u(w_i)$.
- This yields
$$u(w_1) = \underline{u} - \frac{p_{12}}{p_{22} - p_{12}} a_2$$
and
$$u(w_2) = \underline{u} + \frac{1 - p_{12}}{p_{22} - p_{12}} a_2$$
- Now the principal's pay-off can be determined
$$W = p_{22}(x_2 - w_2) + (1 - p_{22})(x_1 - w_1)$$
where $w_2 = u^{-1}\left(\underline{u} + \frac{1-p_{12}}{p_{22}-p_{12}} a_2\right)$ and $w_1 = u^{-1}\left(\underline{u} - \frac{p_{12}}{p_{22}-p_{12}} a_2\right)$.

Moral hazard

- Without specifying the parameter values it is not possible to say when the principal wants the agent to choose a_2 rather than a_1.
- The principal can implement a_1 by wage $w = \underline{u}$.
- If the principal's pay-off from this is less than from the contract that implements a_2 then the principal chooses the contract that implements a_2.