Nöldeke-Schmidt-model

Same as Hart-Moore-model with the assumption that the court can determine whether the seller refused to deliver the good or the buyer refused to accept the delivery.

One can also consider more general randomness with $\omega = (\omega_b, \omega_s)$ on - $= [0, 1]^2$, joint distribution $f(\omega)$,and marginals $f_b(\omega_b)$ and $f_s(\omega_s)$.

Let valuation and cost be $v(\omega_b, \beta)$ and $c(\omega_s, \sigma)$.

The problem of the parties is to design a contract that implements the first best, i.e., maximises

$$W(\beta, \sigma) = \int_0^1 \int_0^1 \left(\max\{v(\omega_b, \beta) - c(\omega_s, \sigma), 0\}\right) f(\omega) d\omega_s d\omega_b - h_b(\beta) - h_s(\sigma) \quad (1)$$

The date-0 contract specifies prices (p_0, p_1) depending on whether the seller did not deliver the good or did do so.

Renegotiation consists of just one instance at which parties $i \in \{b, s\}$ can send an offer $\left(\widetilde{p}_0^i, \widetilde{p}_1^i\right)$.

After (no)trading the parties can present the renegotiation offers to the court. The court observes whether delivery took place, and enforces the original contract unless

1. Exactly one party produces a renegotiation offer signed by the other party specifying different terms of trade,

   or

2. Both parties present identical renegotiation offers signed by the opposite party specifying different terms of trade

in which cases the new contract is enforced.

Proposition 1 Let the date-0 contract be (p_0, p_1). Given β and σ trade is ex-post efficient, and the payment of the buyer to the seller is given by

i) $p = p_0 + qc(\omega_s, \sigma)$ if $p_1 - p_0 \leq c(\omega_s, \sigma)$

ii) $p = p_1 - c(\omega_s, \sigma) + qc(\omega_s, \sigma)$ if $p_1 - p_0 > c(\omega_s, \sigma)$.

The difference to Proposition 1 (Hart-Moore) is that their contract is conditional on q. If $v < c < p_1 - p_0$ HM-contract is not renegotiated and $q = 0$ and $p = p_0$ as the buyer can prevent trade unilaterally.

If $c < v < p_1 - p_0$ HM-contract is renegotiated NS-contract is not.

NS contract can be expressed as (p_0, k) where $k = p_1 - p_0$. Payment p_0 is made always, and k is made if the seller uses the option to deliver the good. In this sense it is an option contract.

Corollary 1 The agents' expected utilities under the option contract are

$$\begin{aligned} U_b(\sigma, \beta, p_0, k) &= -h_b(\beta) - p_0 \\ &\quad + \int_0^1 \int_0^1 \max\{v(\omega_b, \beta) - c(\omega_s, \sigma), 0\} f(\omega) d\omega_s d\omega_b \\ &\quad - \int_0^1 \max\{k - c(\omega_s, \sigma), 0\} f_s(\omega_s) d\omega_s \end{aligned} \tag{2}$$

$$U_s(\sigma, \beta, p_0, k) = -h_s(\sigma) + p_0 + \int_0^1 \max\{k - c(\omega_s, \sigma), 0\} f_s(\omega_s) d\omega_s \tag{3}$$

Proof. By proposition date-2 payoffs are

$$u_b = -h_b(\beta) + \begin{cases} -p_0 \text{ in i) if } v < c \\ v - p_0 - c \text{ in i) if } v \geq c \\ c - p_0 - k \text{ in ii) if } v < c \\ v - p_0 - k \text{ in ii) if } v \geq c \end{cases} \tag{4}$$

$$u_s = -h_s(\sigma) + \begin{cases} p_0 \text{ in i)} \\ p_0 + k - c \text{ in ii)} \end{cases} \tag{5}$$

Denote for instance $\{\omega = (\omega_b, \omega_s) : v(\omega_b, \beta) > c(\omega_s, \sigma)\}$ by $\{v > c\}$. Performing the integration in (5) yields

$$U_b(\sigma, \beta, p_0, k) = -h_b(\beta) - \int_{\{c>k\}} \int_{\{v<c\}} p_0 d\omega_b d\omega_c$$

$$+ \int_{\{c>k\}} \int_{\{v>c\}} (v - p_0 - c)\, d\omega_b d\omega_c + \int_{\{c<k\}} \int_{\{v<c\}} (c - p_0 - k)\, d\omega_b d\omega_c$$

$$+ \int_{\{c<k\}} \int_{\{v>c\}} (v - p_0 - k - c + c)\, d\omega_b d\omega_c$$

Notice that in the last case c has been subtracted and added. Take for example $v - c$ under each integration. It is integrated over the set

$$\{c > k\} \cup \{v > c\} \cup \{c < k\} \cup \{v > c\} = \{v > c\}$$

The rest goes in an analogous way.QED

Whenever there is renegotiation the final price is independent of the buyer's investment decision; it depends only on the seller's cost. Buyer always gets the total surplus minus the final price. Thus, s/he gets full return to his/her investment whenever the parties trade.

The problem is to make the seller invest efficiently; given seller's investment the buyer invests efficiently.

Given (p_0, k) the seller chooses σ to maximise

$$U_s(\sigma, p_0, k) = -h_s(\sigma) + p_0 + \int_0^1 \max\{k - c(\omega_s, \sigma), 0\} f_s(\omega_s) d\omega_s \quad (6)$$

The set of maximisers of (6) is non-empty and depends on k. If k is low enough the seller underinvests, and if it is high enough s/he overinvests, since even for high values of k there is positive probability that trade is inefficient.

Lemma 1 Let $\overline{k} \equiv \max_{\omega_s,\sigma} c\left(\omega_s, \sigma\right)$. Then if $k = 0$ the seller underinvests, and if $k = \overline{k}$ the seller overinvest.

Proposition 2 If there is a unique solution $\sigma(k)$ to (6) for all $k \in \left[0, \overline{k}\right]$ then there exists an option contract (p_0, k) which implements the first best.

Proof By Berge's theorem of maximum (for instance Lucas, Stokey and Green, p. 62) $\sigma(k)$ is continuous, and by the intermediate value theorem there exists k^* such that $\sigma\left(k^*\right) = \sigma^*$. Option contract (p_0, k^*) induces the first best by construction.QED