Location and pricing choices of oligopolies

Models where the consumers face prices, search costs, differentiated products, and firms must decide about the degree of differentiation and prices. Sometimes the consumers observe the prices, sometimes not, sometimes they observe the product qualities, sometimes not.

Wolinsky's (1983) model

Firms and consumers on Salop's circle.

Each consumer has an ideal product, and unit demand.

Utility, $u(y)$, is decreasing in a product's distance, y, from the ideal product.

Consumers do not know which firm sells which product, neither the firms' prices.

Visiting a store costs k.

Firms carry one brand, have constant marginal cost c, and a fixed cost F.

Focus on symmetric equilibrium where all products are uniformly represented, and all firms charge price p^*.

Optimal search strategy is to stop when finding a product with distance $y < R$ where

$$\int_0^R [u(y) - u(R)]\, dy = k$$

If a firm deviates and charges price p a consumer accepts all products $y < r(p)$ where

$$u\left(r(p)\right) - p = u(R) - p^* - k$$

With n firms let the expected demand in equilibrium for firm j with product y_j be $x_j\left(p_j, y_j \left| p^*, n\right.\right)$. Then its profit is

$$\Pi_j\left(p_j, y_j \left| p^*, n\right.\right) = \left(p_j - c\right) x_j\left(p_j, y_j \left| p^*, n\right.\right) - F$$

In equilibrium the following conditions must hold:

1. $\left(p_j, y_j\right) = \left(p_j^*, y_j^*\right) = \arg\max \Pi_j\left(p_j, y_j \left| p^*, n^*\right.\right)$ where y_j^* is j's product in equilibrium.

2. $\Pi_j\left(p_j^*, y_j^* \left| p^*, n^*\right.\right) = 0$, $j \in \{1, ..., n^*\}$.

Proposition 1 For large enough number of consumers there is an equilibrium in which the firms locate symmerically on the circle, and charge the same price.

The aim is to determine when the firms cluster in equilibrium.

For that purpose it is enough to study a situation where n firms are in a single location and one firm is in an isolated location.

The consumer has to decide whether to visit the isolated firm or the cluster first.

Travel costs to a destination are $t_0 + tD$ where D is the distance.

Consumers know the number of stores in a location.

There are n stores in location h_1 and 1 store in location h_2.

The distance to h_1 is D_1 and to h_2 it is D_2.

The distance from h_2 to h_1 is Δ.

It is postulated that the price in each store is p^*.

In h_1 the optimal search strategy is given by R since there are no transportation costs there.

If the consumer goes first to h_2 s/he has a reservation product W; with the transportation cost to h_1 it yields the same expected utility as is achievable in h_1.

W is decreasing in n and $\mathbf{\Delta}$.

Proposition 2 There exists N such that for all $n > N$ there exists distance $\Delta(n)$ such that i) if $\Delta < \Delta(n)$ each consumer goes first to the cluster; ii) $\Delta(n)$ is increasing in n.

Proposition 3 There exists $\overline{S}$ such that if the number of consumers S is larger than $\overline{S}$ there exists distance $D(S)$ with the following property: If there exists a location h that is no further than $D(S)$ from each consumer, there exists an equilibrium in which all stores are at location h.

Note that there are clearly multiple equilibria.

Still somewhat surprising result as the firms are assumed to price compete.

Compare to Diamond (1971).

The Deneckere-Peck (1995) model

Oligopolistic competition under stochastic demand and constrained capacity.

Consumers observe the chosen capacities and prices but not the level of demand.

Only one firm can be visited.

There are n firms.

Firms have constant marginal cost of capacity c.

Firms have constant marginal cost of production $c_0 = 0$.

Aggregate number of consumers θ with density h on $[0, m]$.

The expected value is $E(\theta) = \int_0^m \theta f(\theta) d\theta$.

From a (living) consumer's point of view the density of aggregate demand is given by

$$g(\theta) = \frac{\theta f(\theta)}{E(\theta)} \tag{1}$$

Timing: Each firm i chooses capacity k_i, and price p_i. Then nature chooses the level of demand.

Then consumers choose the firm to visit.

Then firms observe demand and produce (up to capacity).

Consumers observe the prices and capacities before they make their choice.

Each consumer has a unit demand and values the good at $v > c$.

Let s be the probability of consuming. Utility at price p is given by

$$U(p, s) = s(v - p) \tag{2}$$

Focus on equilibrium in which all consumers choose same mixed strategy $q = (q_1, ..., q_n)$.

Informally (and incorrectly) leaning on the law of large numbers, it is postulated that the proportion of consumers visiting firm i is q_i.

A consumer who visits firm i expects to get a good with probability

$$s\left(k_i/q_i\right) \equiv s(K_i) = \int_0^{K_i} 1 \cdot \frac{\theta f(\theta)}{E(\theta)} d\theta + \int_{K_i}^{m} \frac{K_i}{\theta} \frac{\theta f(\theta)}{E(\theta)} d\theta$$

This is equivalent to

$$s(K_i) = \int_0^{m} \frac{\theta f(\theta)}{E(\theta)} d\theta - \int_{K_i}^{m} \frac{\theta f(\theta)}{E(\theta)} d\theta + \int_{K_i}^{m} \frac{K_i}{\theta} \frac{\theta f(\theta)}{E(\theta)} d\theta \qquad (3)$$

which is equivalent to

$$s(K_i) = 1 - \frac{\int_{K_i}^{m} \left(\theta - K_i\right) f(\theta) d\theta}{E(\theta)} \qquad (4)$$

$s(K_i)$ is increasing as $s'(K_i) = \frac{1 - F(K_i)}{E(\theta)} > 0$.

$s(K_i)$ is concave as $s''(K_i) = \frac{-f(K_i)}{E(\theta)} < 0$.

Consumers observe prices $p = (p_1, ..., p_n)$ and capacities $k = (k_1, ..., k_n)$.

They choose strategy $(q_1, ..., q_n)$ so that they get the same utility from each firm i such that $q_i > 0$

$$(v - p_i)s(K_i) = U^*(p, k) \tag{5}$$

Notice that if a firm i lowers price q_i increases just a little.

Firm i maximises

$$\pi_i = p_i \int_0^{K_i} q_i \theta f(\theta) d\theta + p_i \int_{K_i}^{m} k_i f(\theta) d\theta - c k_i \tag{6}$$

$$\pi_i = p_i E(\theta) q_i s(K_i) - c k_i \tag{7}$$

Let K^* be defined by $s'(K^*) = \frac{c}{vE(\theta)}$.

Lemma 2 If firms choose pure strategies, all active firms choose $K_i = K^*$.

Proof. Suppose that for firm i $K_i \neq K^*$, and consider deviation $k_i' = q_i K^*$ and $p_i' = v - \frac{U^*}{s(K^*)}$. This yields the utility U^* if $q_i' = q_i$, and because there is unique q that solves the consumers' problem (needs separate proof) $q_i' = q_i$ indeed. Inserting p_i' into (7) yields

$$\pi_i'/q_i = \mu v s(K^*) - cK^* - \mu U^* \tag{8}$$

Solving p_i from (5) and inserting in (7) yields profit in postulated equilibrium

$$\pi_i/q_i = \mu v s(K_i) - cK_i - \mu U^* \tag{9}$$

As a function of K the right hand side above is concave and it is uniquely maximised at $K = K^*$. Thus, there exists a profitable deviation.QED

Lemma 3 In a pure strategy equilibrium firms make positive profits.

Proof. Suppose not. Then they make zero profits. Since in equilibrium $s(K^*) < 1$ a firm can raise its price a little, lower its capacity a little such that the service rate remains the same. This means that it gets the same number of consumers but at a higher price. This is a profitable deviation.QED

Lemma 4 In a pure strategy equilibrium all firms are active.

Proof. An existing firm can increase its capacity a little without profits going to zero. But this is equivalent to an inactive firm entering the market with small capacity.QED

Proposition 1 If an equilibrium in pure strategies exists it is unique, and given by

$$k_i = K^*/n = F^{-1}(1 - c/v)/n \tag{10}$$

$$p_i = \frac{cK^*n/(n-1)}{\mu s(K^*) + cK^*/(v(n-1))} \tag{11}$$

$$q_i = 1/n \tag{12}$$

Proof. Since all firms choose the same service rate they must choose the same price p, and consequently the consumers visit the firms with the same probability. Firm i's problem is

$$\max_{p_i,k_i} q_i p_i E(\theta) s(k_i/q_i) - ck_i \tag{13}$$

subject to

$$(v-p_i)s(k_i/q_i)=(v-p)s(k_j/q_j) \tag{14}$$

for all $j \neq i$. Solve for p_i from the constraint, and notice that in a symmetric equilibrium $k_j/q_j = k_{-i}/(1-q_i)$, to get problem

$$\max_{q_i,k_i} q_i E(\theta) s\left(k_i/q_i\right) v-(v-p) s\left(\frac{k_{-i}}{1-q_i}\right)-c k_i \tag{15}$$

Now FOC$_{k_i}$ implies that $s'\left(k_i/q_i\right)=\frac{c}{E(\theta)v}$.

By definition before Lemma 2 this means that $k_i/q_i = K^*$, and by Lemmas 2 and 4 also $\frac{k_{-i}}{1-q_i}=K^*$.

FOC$_{q_i}$: $vs\left(K^*\right)=vs'\left(K^*\right)K^*+(v-p)s'\left(K^*\right)K^*\frac{q_i}{1-q_i}+(v-p)s\left(K^*\right)$. This holds for all i; the only place where there is i is $\frac{q_i}{1-q_i}$, and thus $q_i=1/n$. Plugging this data into FOC$_{q_i}$ and solving yields the desired result.QED

Everything above is conditional on 'if there exists an equilibrium where firms use pure strategies'. Existence depends on n. Necessity (the easy part) basically consists of the derivation of FOCs like above. It is advisable to check what sufficiency requires.

Model of Dudey (1990)

The essentials of the idea are conveyed by the following simple example.

There are m consumers with inverse demand $p = 1 - q$ for each.

There are n firms with constant marginal cost $c \geq 0$ technology.

There is a large number ($> n$) of locations where firms may go.

There are no travelling costs.

Consumers find out about prices when they visit a store.

Timing:

1. Firms choose locations.
2. Consumers observe the choices.
3. 3. Consumers decide where to go.
4. 4. Firms Cournot-compete knowing how many consumers are in a location.
5. 5. Consumers buy.

Assume that l consumers arrive to a monopoly.

Inverse demand is then $p = 1 - \frac{q}{l}$.

Monopoly maximises $\pi = q\left(1 - \frac{q}{l} - c\right)$.

Solution $q^M = \frac{l(1-c)}{2}$.

Price $p^M = \frac{1+c}{2}$ is independent of l.

Assume that l consumer arrive at a location with r firms.

Firm i maximises $\pi_i = q_i \left(1 - \frac{q_i}{l} - \frac{q_{-i}}{l} - c\right)$.

Equilibrium quantity $q_i^C = \frac{l(1-c)}{r+1}$.

Price $p^C = \frac{r+c}{r+1}$ is independent of l.

Consumers care about price and $p^M \geq p^C$.

Proposition If there are at least three firms, there is an equilibrium where all the firms cluster.