Kultti (2003) model

There are B buyers each with unit demand and unit valuation.

There are S sellers each with unit supply and zero valuation.

Infinite horizon, discrete time economy.

Sellers may choose location which buyers visit.

If there are l location the number of sellers in location i is s_i.

Buyers use symmetric mixed strategy $\beta = (\beta_1, ..., \beta_l)$.

The number of sellers expected at location i is distributed by $Bin\left(\beta_i, B\right)$.

If in any location there are at least as many sellers as buyers each buyer makes a take-it-or-leave-it offer to some seller.

If in any location there are more buyers than sellers the buyers engage in an auction to get the objects.

If there are more sellers than buyers the sellers do not want to go to a same location.

Definition 2. A Walrasian market is a configuration where the sellers go to the same location.

Result 1. Assume that $S \geq B$. Then the Walrasian market is not an equilibrium.

Result 2. Assume that $S \geq B$. Then in equilibrium, in any location i, $s_i < B$.

Consider a large market where S and B approach infinity such that B/S remains constant.

If there are n sellers in each location the number of buyers expected in any location is distributed by $Poisson(n\alpha)$ where $\alpha = B/S$.

Assume from now on that $\alpha = B/S < 1$.

Focus on a steady state equilibrium where those who trade exit and are replaced by identical agents.

A seller's life time utility is given by

$$V = \delta \left\{ e^{-n\alpha} \sum_{k=0}^{n} \frac{(n\alpha)^k}{k!} V + \left(1 - e^{-n\alpha} \sum_{k=0}^{n} \frac{(n\alpha)^k}{k!} \right) (1 - W) \right\} \qquad (1)$$

A buyer's life time utility is given by

$$W = \delta \left\{ e^{-n\alpha} \sum_{k=0}^{n-1} \frac{(n\alpha)^k}{k!} (1 - V) + \left(1 - e^{-n\alpha} \sum_{k=0}^{n-1} \frac{(n\alpha)^k}{k!} \right) W \right\} \qquad (2)$$

For economy denote the CDF of a $Poisson(\lambda)$ by

$$F_\lambda(n) = e^{-\lambda} \sum_{k=0}^{n} \frac{(\lambda)^k}{k!} \tag{3}$$

Solving for V and W

$$V = \delta \frac{1 - F_{n\alpha}(n)}{1 - \delta (n\alpha)^n / n!} \tag{4}$$

$$W = \delta \frac{F_{n\alpha}(n-1)}{1 - \delta (n\alpha)^n / n!} \tag{5}$$

The lower $F_{n\alpha}(n)$ the higher the sellers' utility.

Since sellers get to choose locations the aim is to determine n that maximises the sellers' utility.

Result 3. Assume that $\alpha = 1$, and that there are n sellers per location. When δ is close to zero regrouping so that there are $n + 1$ sellers per location increases the sellers' utility.

Proof. The claim is equivalent to $F_{n+1}(n+1) < F_n(n)$. Using the fact that $Poisson(\lambda)$-distribution function can be expressed as $F_\lambda(h) = \frac{1}{h!}\int_\lambda^\infty y^h e^{-y} dy$ the above condition becomes

$$\int_{n+1}^{\infty} y^{n+1} e^{-y} dy < (n+1) \int_n^\infty y^n e^{-y} dy$$

$$= -n^{n+1} e^{-n} + \int_n^{n+1} y^{n+1} e^{-y} dy + \int_{n+1}^{\infty} y^{n+1} e^{-y} dy$$

which is equivalent to

$$n^{n+1} e^{-n} < \int_n^{n+1} y^{n+1} e^{-y} dy$$

The integrand on the right hand side is increasing in $[n, n+1]$ and reaches its maximum at point where

$$(n+1) y^n e^{-y} - y^{n+1} e^{-y} = 0$$

that is at $y = n+1$.

The integrand is then at least $n^{n+1}e^{-n}$.QED

Increasing the number of sellers makes competition for buyers more severe, and one would expect that the sellers are more dispersed.

Define $\alpha_n \equiv n\,[\log(n+1) - \log n]$. Notice that α_n is increasing in n.

Result 4. When $\alpha_n > \alpha \geq \alpha_{n-1}$ the equilibrium market structure is such that there are n sellers in a location.