# The topic of the talk: About firms' tendency to cluster

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Why do firms locate close to each other?

What is the equilibrium market structure?

Petrol stations, shopping centres, car dealers, furniture outlets.

With symmetric firms and constant marginal costs Bertrand competition leads to zero profits.

To explain the phenomenon the standard setting must be altered.

Three non-standard assumptions.

1. The firms are capacity constrained.

2. The demand is stochastic; firms learn about it only after pricing decisions.

3. Location matters.

With limited capacity and stochastic demand firms face a trade-off between locating separately and clustering.

If firms are clustered they compete fiercely when demand is less than supply, and they act as monopolists if demand exceeds supply.

If firms are located separately they compete for customers even when demand exceeds supply.

With uncertain demand these features of competition manifest in 'expectation'.

If it were known that demand exceeds supply the firms would cluster.

If it were known that demand falls short of supply the firms would locate separately.

The aim is to demonstrate how this trade-off affects location, pricing and profits.

Rest of the talk: A short literature review, an example, general result, shortcomings and bad modelling choices and lack of relevance and open questions.

# Some somewhat related literature

Stahl (1983) and Wolinsky (1983) study location choice when consumers do not know the prices. Because of search costs clustering may be a profitable strategy.

In Dudey (1990) consumers do not observe prices but pricing is modelled carefully. The consumers expect higher degree of competition and lower prices in clusters, and thus clusters attract more consumers than sellers who are separated.

Dana (1993) recognises the importance of demand uncertainty but his interest is in price dispersion. Capacity is costly, the sellers set a menu of prices before the demand is known.

Deneckere and Peck (1995) study a situation with a finite number of non-clustered firms and a continuum of buyers. Demand is uncertain, and the firms choose capacity. Finite number of firms gets rid of part of the uncertainty about the number of buyers. Deneckere and Peck do not address the location choices of the firms at all.

Burdett, Shi and Wright (2001) study the equilibrium price posting in a finite agent deterministic world that corresponds to the non-clustered market of my model. They shortly study the effect of capacity but not in equilibrium.

A complete solution to firm location when prices are determined by auction, rather than price posting, and when there are fewer buyers than sellers is given in Kultti (2003a).

In the older literature already Chamberlin (1933) realised the trade-off between increased competition from locating close to each other and the positive effect this has on attracting consumers.

# An example

Two identical firms 1 and 2.

Both possess one unit of a good.

With probability $p_1$ there is one buyer, with probability $p_2$ two buyers and with probability $1 - p_1 - p_2$ three buyers.

Two different market structures.

1. Non-clustered market: Firms physically separated, buyers can visit only one of them. The firms post prices to attract buyers.

2. Clustered market: Firms are located in the same place, buyers can visit both of them. The firms again post prices to attract buyers.

In the non-clustered market there is a unique symmetric equilibrium in pure strategies.

In the clustered market the unique symmetric strategy is a mixed strategy.

## Non-clustered market

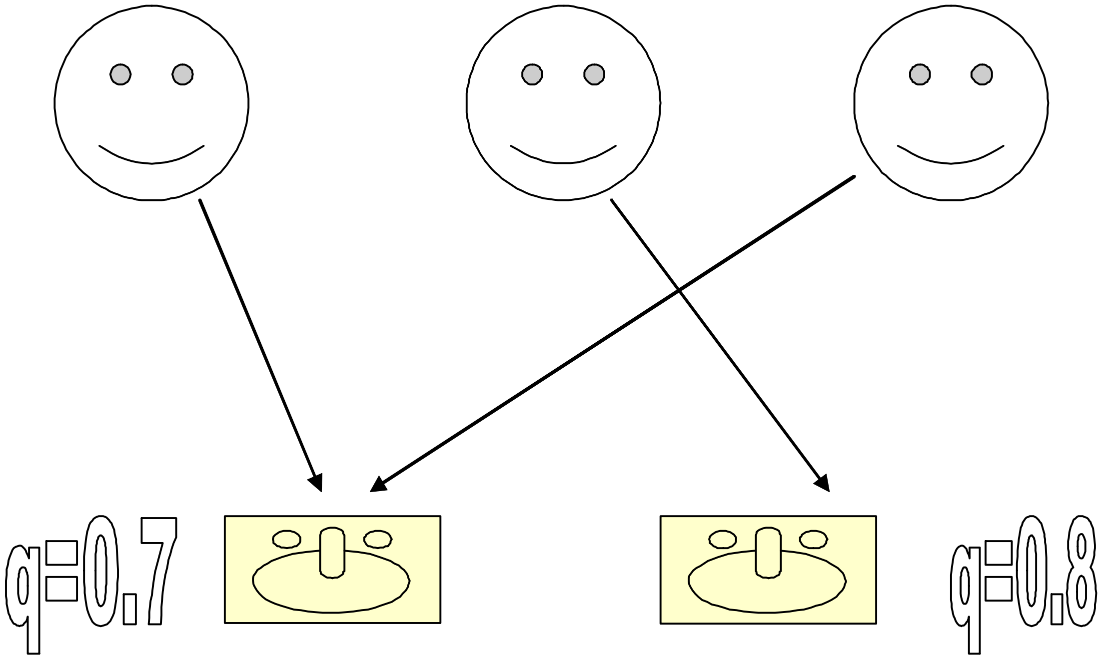


Figure 1:

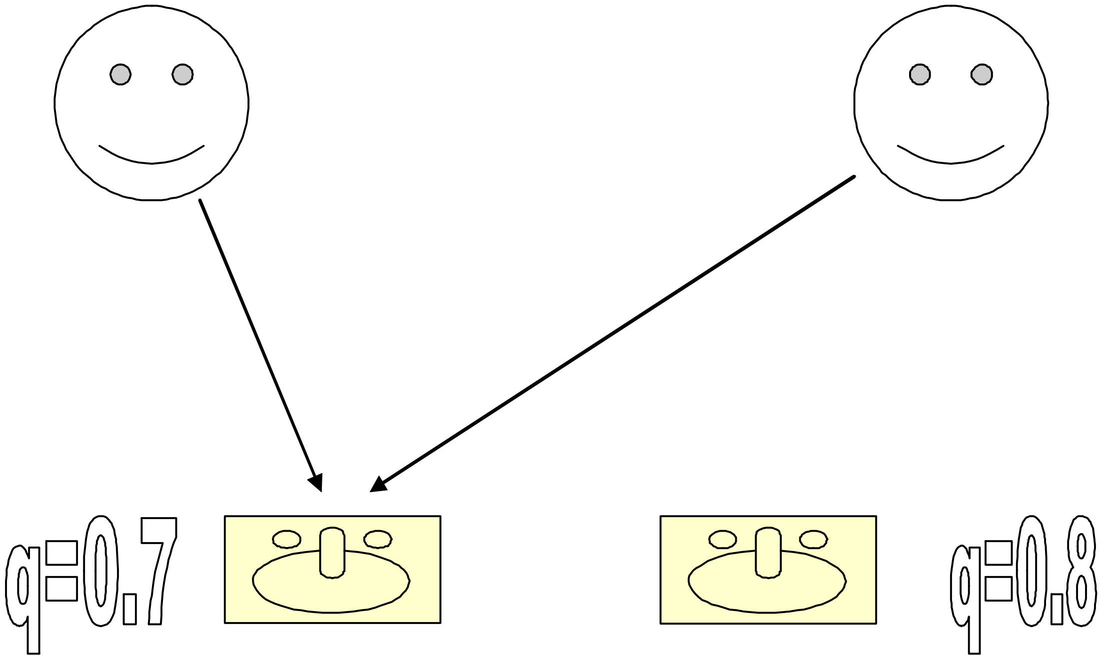


Figure 2:

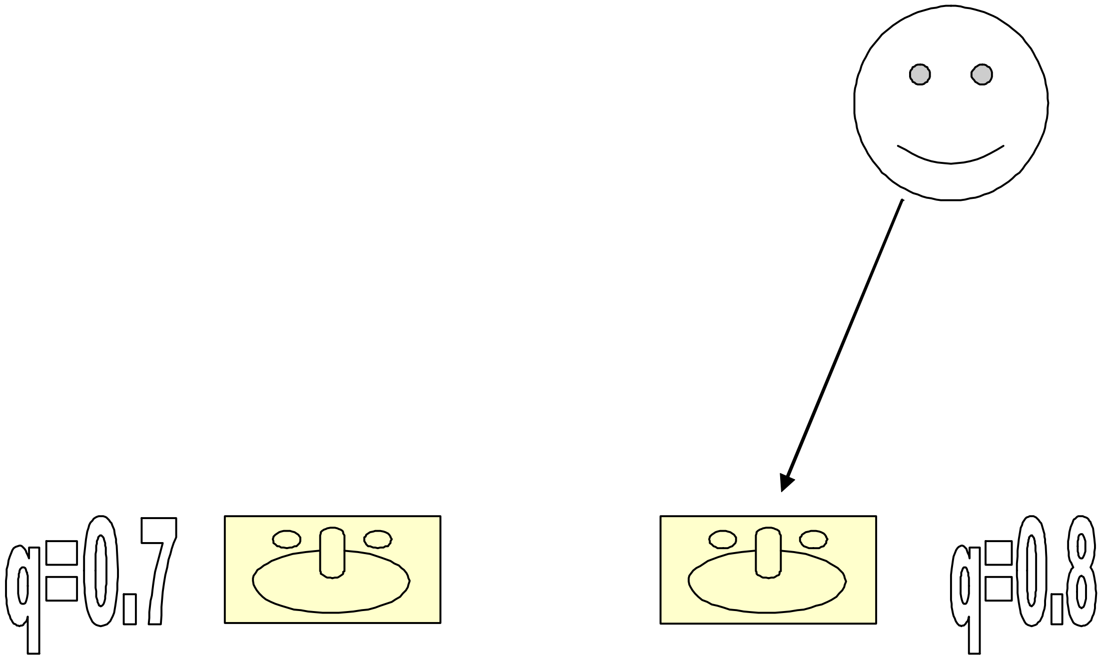


Figure 3:

Price of firm 1 $q_1$ and that of firm 2 $q_2$.

Probability that a buyer goes to firm 1 $\pi_1$.

A buyer must assess the probability that the demand is one, two and three given the information that he (the buyer) exists.

$$r_1 = \frac{\frac{1}{3}p_1}{\frac{1}{3}p_1 + \frac{2}{3}p_2 + (1 - p_1 - p_2)} = \frac{p_1}{3 - 2p_1 - p_2}$$

$$r_2 = \frac{2p_2}{3 - 2p_1 - p_2}$$

$$r_3 = \frac{3\,(1 - p_1 - p_2)}{3 - 2p_1 - p_2}$$

Expected utility of visiting firm 1

$$r_1(1-q_1) + r_2\frac{\pi_1}{2}(1-q_1) + r_2(1-\pi_1)(1-q_1) + \tag{1}$$
$$r_3\frac{\pi_1^2}{3}(1-q_1) + r_3\frac{2\pi_1(1-\pi_1)}{2}(1-q_1) +$$
$$r_3(1-\pi_1)^2(1-q_1)$$

Expected utility of visiting firm 2

$$r_1(1-q_2) + r_2\pi_1(1-q_2) + r_2\frac{1-\pi_1}{2}(1-q_2) + \tag{2}$$
$$r_3\pi_1^2(1-q_2) + r_3\frac{2\pi_1(1-\pi_1)}{2}(1-q_2) +$$
$$r_3\frac{(1-\pi_1)^2}{3}(1-q_2)$$

Equating (1) and (2) determines $\pi_1$.

In symmetric equilibrium $q_1 = q_2 = q$, and $\pi_1 = \frac{1}{2}$.

Totally differentiating the equality (1)=(2) and inserting the equilibrium conditions yields

$$\frac{\partial \pi_1}{\partial q_1} = -\frac{12r_1 + 9r_2 + 7r_3}{(1-q)\,[12r_2 + 16r_3]}$$

Firm 1's problem is

$$\max_{q_1} p_1\pi_1 q_1 + p_2\left(1-(1-\pi_1)^2\right)q_1 +$$

$$(1-p_1-p_2)\left(1-(1-\pi_1)^3\right)q_1$$

Evaluating the first order condition of this problem at the symmetric equilibrium where $q_1 = q_2 = q$ and $\pi_1 = \frac{1}{2}$ yields

$$q = \frac{(12r_2 + 16r_3)(4p_1 + 6p_2 + 7p_3)}{(12r_2 + 16r_3)(4p_1 + 6p_2 + 7p_3) + (12r_1 + 9r_2 + 7r_3)(8p_1 + 8p_2 + 6p_3)}$$

The expected utility of a firm is

$$q\frac{4p_1 + 6p_2 + 7p_3}{8}$$

**Clustered market**

As long as $1 > p_1 > 0$ the firms' pricing is in mixed strategies.

Denote this strategy by $F$ and assume that its support is a closed interval $[l, L]$.

It is immediate that $L = 1$.

The firm quoting price $L = 1$ makes a sale with probability $1 - p_1$ and this is also its expected utility.

The firm quoting price $l$ makes a sale for certain and thus $l = 1 - p_1$.

If a firm posts price $\rho \in (l, L)$ its expected utility is

$$p_1 \left(1 - F(\rho)\right) \rho + \left(1 - p_1\right) \rho$$

This choice, too, must yield utility $1 - p_1$ and thus

$$F(\rho) = \frac{\rho - 1 + p_1}{\rho p_1}$$

**Proposition** Whenever $1 > p_1 > 0$ the expected utility of a firm is higher in the clustered market than in the non-clustered market.

When the firms are not clustered the ex-ante probability of ending up with no buyer when pricing symmetrically is $\frac{1}{2}p_1 + \frac{1}{4}p_2 + \frac{1}{8}(1 - p_1 - p_2)$,

while when they are clustered it is $\frac{1}{2}p_1$.

# GENERAL CASE

**Infinite number of firms and buyers**

Measure of firms unity.

Measure of buyers, $\theta$, distribution $H$ on an interval $[0, m]$, $m > 1$.

A buyer's expectation that there are exactly $a \in [0, m]$ buyers

$$g(a) = \frac{\frac{a}{m}h(a)}{\int_0^m \frac{x}{m}h(x)dx} = \frac{ah(a)}{E(\theta)}$$

*The timing*: First firms simultaneously quote prices, then buyers observe the prices and based on these they simultaneously approach the firms.

Focus on symmetric equilibrium; in the clustered market pricing in mixed strategies, while in the non-clustered market pricing in pure strategies.

**Definition: Equilibrium** A symmetric equilibrium in a particular market consists of symmetric pricing strategies of the firms, and symmetric contact strategies of the buyers such that any firm's strategy is the best response to the other firms' and buyers' strategies, and any buyer's strategy is the best response to the firms' strategies and other buyers' strategies.

**Non-clustered market**

Given the unknown number $\theta$ of buyers, the number of buyers that contacts a firm is Poisson-distributed with parameter

$\frac{\#buyers}{\#firms} = \frac{\theta}{1} = \theta.$

The probability of exactly $k$ buyers is $e^{-\theta}\frac{\theta^k}{k!}$.

Equilibrium price $q$.

To determine $q$ assume that proportion $\varepsilon$ of the firms deviate to $\widetilde{q}$.

In equilibrium $\widetilde{q} = q$.

When $\varepsilon$ approaches zero one gets the limit of the equilibrium price in the finite agent model.

With probability $z$ a buyer goes to a deviating firm and with probability $1 - z$ to a non-deviating firm.

The Poisson-rate for non-deviating firms

$$\alpha = \frac{1-z}{1-\varepsilon}\theta$$

and for deviating firms

$$\beta = \frac{z}{\varepsilon}\theta$$

The probability that a buyer gets a good

$$\sum_{i=0}^{\infty} e^{-x}\frac{x^i}{i!}\frac{1}{i+1} = \frac{1-e^{-x}}{x}$$

$x \in \{\alpha, \beta\}$

The utility of going to a non-deviating firm

$$\int_0^m (1-q)\frac{1-e^{-\alpha}}{\alpha} g(\theta) d\theta$$

and to a deviator

$$\int_0^m (1-\widetilde{q})\frac{1-e^{-\beta}}{\beta} g(\theta) d\theta$$

In equilibrium, these have to be equal which condition determines $z$.

We need

$$\frac{dz}{d\widetilde{q}} =$$

$$-\frac{\int_0^m \frac{1-e^{-\beta}}{\beta} g(\theta) d\theta}{\int_0^m (1-q) \frac{1-e^{-\alpha}-\alpha e^{-\alpha}}{\alpha^2} \frac{\theta}{1-\varepsilon} g(\theta) d\theta + \int_0^m (1-\widetilde{q}) \frac{1-e^{-\beta}-\beta e^{-\beta}}{\beta^2} \frac{\theta}{\varepsilon} g(\theta) d\theta}$$

The deviating firm

$$\max_{\widetilde{q}} \int_0^m \widetilde{q}\left(1 - e^{-\beta}\right) h(\theta) d\theta$$

The first order condition

$$\int_0^m \left(1 - e^{-\beta} + \widetilde{q} e^{-\beta} \frac{\theta}{\varepsilon} \frac{dz}{d\widetilde{q}}\right) h(\theta) d\theta = 0$$

In a symmetric Nash equilibrium $\widetilde{q} = q$ and $\alpha = \beta = \theta$.

Inserting these data into the FOC and letting $\varepsilon$ approach zero

$$q = \frac{\int_0^m \left(1 - e^{-\theta} - \theta e^{-\theta}\right) h(\theta) d\theta}{\int_0^m \left(1 - e^{-\theta}\right) h(\theta) d\theta}$$

## Clustered market

Firms' equilibrium pricing strategy is mixing.

If $F$ is the mixed strategy on $[l, L]$ then $L = 1$, $F$ has no atoms and no gaps, and $l = 1 - H(1)$.

**Expected profits**

Profits in the clustered market $1 - H(1)$.

Profits in the non-clustered market $\int_0^m \left(1 - e^{-\theta}\right) qh(\theta)d\theta$.

The former is greater than the latter if

$$e^{-m}(1+m) + \int_0^m \theta e^{-\theta} H(\theta)d\theta > H(1)$$

**Proposition** If the probability that there are at most as many buyers as sellers, here unity, is sufficiently small, i.e.,

$$e^{-m}(1+m)+\int_0^m \theta e^{-\theta} H(\theta) d\theta > H(1)$$

then the firms fare better in the clustered market than in the non-clustered market.

If $H$ is uniform on $[0, m]$ the condition becomes

$$e^{-m}(2+m) < 1$$

and this is satisfied for all $m > 1.11$.

# EQUILIBRIUM DEGREE OF CLUSTERING

So far only the buyers' responses have been considered.

**Definition: Equilibrium** An equilibrium consists of firms' choice of market, the firms symmetric pricing strategies in each market, the buyers choice of the market, and the symmetric contact strategies of the buyers in each market such that any firm's strategy is the best response to the other firms' and buyers' strategies, and any buyer's strategy is the best response to the firms' strategies and other buyers' strategies.

**Non-clustered market**

Proportion of firms in the non-clustered market $\sigma$.

Proportion of buyers in the non-clustered market $z$.

Poisson-rate governing the meetings is $\gamma = \frac{z\theta}{\sigma}$.

Equilibrium price

$$q = \frac{\int_0^m \left(1 - e^{-\gamma} - \gamma e^{-\gamma}\right) h(\theta) d\theta}{\int_0^m \left(1 - e^{-\gamma}\right) h(\theta) d\theta}$$

Expected utility of a buyer

$$\int_0^m \frac{1-e^{-\gamma}}{\gamma} g(\theta) d\theta (1-q)$$

$$= \int_0^m \frac{1-e^{-\gamma}}{\gamma} g(\theta) d\theta \frac{\int_0^m \gamma e^{-\gamma} h(\theta) d\theta}{\int_0^m \left(1-e^{-\gamma}\right) h(\theta) d\theta}$$

$$= \frac{1}{E(\theta)} \int_0^m \theta e^{-\gamma} h(\theta) d\theta$$

## Clustered market

Proportion of firms in the clustered market $1 - \sigma$.

Proportion of buyers in the clustered market $1 - z$.

Mixed strategy $F$ with support $\left[1 - H(\frac{1-\sigma}{1-z}), 1\right]$ and $F(1) = 1 - \sigma$.

$F(q)$ is determined by

$$\left(1 - H\left(\frac{F(q)}{1-z}\right)\right) q = 1 - H\left(\frac{1-\sigma}{1-z}\right)$$

Explicit expression

$$F(q) = (1-z) H^{-1}\left(1 - \frac{1 - H\left(\frac{1-\sigma}{1-z}\right)}{q}\right)$$

If $(1-z)\theta < 1-\sigma$ each buyer gets a good.

If $(1-z)\theta \geq 1-\sigma$ then the buyers are rationed.

Let $\omega$ be the highest price at which trading takes place.

It is given by $F(\omega) = \min\{1-\sigma, (1-z)\theta\}$.

A buyer's utility

$$\int_0^{(1-\sigma)/(1-z)} g(\theta) \int_{1-H\left(\frac{1-\sigma}{1-z}\right)}^{F^{-1}((1-z)\theta)} (1-q)\frac{f(q)}{1-\sigma} dq d\theta$$

$$+ \int_{(1-\sigma)/(1-z)}^{m} g(\theta)\frac{1-\sigma}{\theta(1-z)} \int_{1-H\left(\frac{1-\sigma}{1-z}\right)}^{1} (1-q)\frac{f(q)}{1-\sigma} dq d\theta$$

This is too complicated.

Assume that each firm charges price $p$.

A firm's expected utility

$$\int_0^{(1-\sigma)/(1-z)} \frac{\theta(1-z)}{1-\sigma} p h(\theta) d\theta + \int_{(1-\sigma)/(1-z)}^{m} p h(\theta) d\theta$$

Forcing this to equal $1 - H\left(\frac{1-\sigma}{1-z}\right)$ and solve $p$

$$p = \frac{1 - H\left(\frac{1-\sigma}{1-z}\right)}{1 - \int_0^{(1-\sigma)/(1-z)} \frac{1-z}{1-\sigma} H(\theta) d\theta}$$

Since the total number of trades is the same under mixed strategy $F$ and under the scenario where the firms charge $p$ the buyers' expected utility has to be the same, too, under the two scenarios. Price $p$ yields a buyer

$$\int_0^{(1-\sigma)/(1-z)}(1-p)g(\theta)d\theta+\int_{(1-\sigma)/(1-z)}^{m}\frac{1-\sigma}{\theta(1-z)}(1-p)g(\theta)d\theta$$

$$= \frac{1}{E(\theta)}\left[\frac{1-\sigma}{1-z}H\left(\frac{1-\sigma}{1-z}\right)-\int_0^{(1-\sigma)/(1-z)}H(\theta)d\theta\right]$$

If there is an equilibrium where some firms are in the clustered and some in the non-clustered market, the buyers must fare equally well in both markets

$$\left[\frac{1-\sigma}{1-z} H\left(\frac{1-\sigma}{1-z}\right) - \int_0^{(1-\sigma)/(1-z)} H(\theta) d\theta\right] = \int_0^m \theta e^{-\gamma} h(\theta) d\theta$$

This is equivalent to

$$\int_0^{(1-\sigma)/(1-z)} \theta h(\theta) d\theta = \int_0^m \theta e^{-\gamma} h(\theta) d\theta$$

**Lemma** When the buyers are indifferent between the markets, the firms in the clustered market fare better than the firms in the non-clustered market or

$$\int_0^m (e^{-\gamma} + \gamma e^{-\gamma}) h(\theta) d\theta > H\left(\frac{1-\sigma}{1-z}\right)$$

**Proof**

$$\int_0^m e^{-\gamma} h(\theta) d\theta + \int_0^{(1-\sigma)/(1-z)} \frac{z}{\sigma} \theta h(\theta) d\theta \quad > \quad H\left(\frac{1-\sigma}{1-z}\right) =$$

$$\int_0^{(1-\sigma)/(1-z)} h(\theta) d\theta$$

This is equivalent to

$$\int_0^{(1-\sigma)/(1-z)} \left(\frac{z}{\sigma}\theta - 1 + e^{-\frac{z}{\sigma}\theta}\right) h(\theta)d\theta + \int_{(1-\sigma)/(1-z)}^{m} e^{-\gamma} h(\theta)d\theta > 0$$

which certainly holds as the first integrand is of the form $x-1+e^{-x} \geq 0$.■

**Proposition** There are two equilibria in the model, namely one where all the firms are clustered, and one where all the firms are non-clustered. In particular, in equilibrium the two markets do not co-exist.

**Proposition** The clustered market is the unique perfect equilibrium.

# CAVEATS

- **Exogenous capacity**

Assume free entry of the firms.

Determine welfare and socially optimal number of firms under both market structures with free entry.

**Clustered market**

Number of firms $n_C$.

Entry cost $\xi$.

Expected profit of an entering firm $1 - H(n_C) - \xi$.

Expected number of transactions

$$\int_0^{n_C} \theta h(\theta) d\theta + n_C \int_{n_C}^{m} h(\theta) d\theta = n_C - \int_0^{n_C} H(\theta) d\theta$$

Welfare

$$n_C - \int_0^{n_C} H(\theta)d\theta - n_C\xi$$

Socially optimal number of firms determined by

$$1 - \xi - H(n_C) = 0$$

This is the same as the free entry condition.

**Non-clustered market**

Number of firms $n_N$

Poisson rate governing the meetings $\phi \equiv \frac{\theta}{n_N}$.

Expected profit of an entering firm

$$\int_0^m \left(1 - e^{-\phi} - \phi e^{-\phi}\right) h(\theta) d\theta - \xi$$

Number of transactions

$$\int_0^m n_N \left(1 - e^{-\phi}\right) h(\theta) d\theta$$

Welfare

$$\int_0^m n_N \left(1 - e^{-\phi}\right) h(\theta) d\theta - n_N \xi$$

Socially optimal number of firms determined by

$$\int_0^m \left(1 - e^{-\phi} - \phi e^{-\phi}\right) h(\theta) d\theta - \xi = 0$$

Same as the free-entry condition.

## Comparison

Inserting the free-entry conditions to the expressions for welfare yields in the clustered market

$$n_C H\left(n_C\right) - \int_0^{n_C} H(\theta) d\theta$$

and in the non-clustered market

$$n_N \int_0^m \phi e^{-\phi} h(\theta) d\theta$$

- **Degree of clustering**

- **Commitment**

# CONCLUSION

I determine the equilibrium market structure when firms may cluster or be in separate locations.

Firms compete in prices and buyers observe them before deciding which market/firm to go.

In the clustered market all possible trades are completed.

In the non-clustered market firms must compete for buyers even when demand exceeds supply.

Surprisingly this means that competition is less fierce in the clustered market.

With uncertain demand and capacity constraints the clustered market emerges as the unique perfect equilibrium.