Experimental results of strategic behaviour

- Perhaps the most popular game in experiments has been the ultimatum bargaining game.
- A cake/pie of fixed value is to be divided between two players.
- P1 makes a proposal $(x, 100\% - x)$ about the division.
- P2 either accepts or rejects.
- In the latter case neither player gets anything.
- There is a 'unique' subgame perfect equilibrium; there is a huge number of Nash-equilibria.

- Guth, Werner, R. Schmittberger and B. Schwarz (1982): \An Experimental Analysis of Ultimatum Bargaining," Journal of Economic Behavior and Organization, 3, 367-388, vary the size of the cake and show that players choose on average $x = 63\%$, the mode being $x = 50\%$.
- The players do not offer the opponent anything less than $1DM$.
- Other experiments have produced roughly similar results, namely deviation from the subgame perfect equilibrium.

- In a dictator-game P1 just makes a decision on the allocation; P2 does not have the option to reject.
- One hypothesis is that there is no difference between the ultimatum bargaining and the dictator game.
- There is a big difference; in the dictator game the dictator gets on average 80%.

- One of the basic tenets of decision theory is that people should not choose dominated strategies.
- In game theory the standard assumption is that the game is common knowledge, i.e., people know the structure of the game and they know that other people know the structure of the game, and they know that other people know that they know that other people know the structure of the game and so on.
- Consequently, if people do not choose dominated strategies then other people know this and iteratively proceeding dominated strategies should be eliminated.
- N players choose a number $x \in [0, 100]$.
- The average of the numbers is $\sum_{i=1}^{N} x_i / N = \overline{x}$.
- Let $p \in (0, 1)$.

- The player whose number is closest to $p\bar{x}$ gets a prize and others get nothing.
- Iteratively eliminating dominated strategies gives that choosing any number greater than $p \cdot 100$ does not make sense.
- After one round of thinking/elimination people should infer that choosing any number greater than $p^2 \cdot 100$ does not make sense.
- Eventually, one must conclude that the only choice that survives elimination of dominated strategies is 0.
- In experiments people never choose zero.
- The mode seems to indicate one or two round of iterated dominance.
- When the game is repeated the mode seems to move towards zero.

Randomised strategies

- In section 7 Schelling discusses about random strategies.
- They are very important and actually the existence results of Nash-equilibrium are based on extending the strategy set to include randomised strategies.
- A randomised strategy is a lottery where a player chooses amongst (pure) strategies with probability.
- In the battle of the sexes game, for instance, one randomised strategy would be $(\frac{1}{3}, Bo; \frac{2}{3}, Ba)$, i.e., a player chooses boxing with probability one third and ballet with probability two thirds.

- There is some disagreement about what randomised strategies mean.
- The most straightforward way is to assume that players actually use some randomising device.
- A more fruitful view is that the random elements depict the uncertainty of other players about a particular player's choice.
- Let us actually determine some equilibrium randomised strategies for some games.

- A prime example of a game with no Nash-equilibrium in pure strategies is the matching pennies game

	H	T
H	$1, -1$	$-1, 1$
T	$-1, 1$	$1, -1$

- One would be pretty surprised if the randomised equilibrium strategy would not be to make either choice with equal probability.
- The basic insight is the following: In equilibrium, if everyone is using his/her equilibrium strategy (perhaps mixed) a player who should use a mixed strategy in equilibrium must be indifferent between the choices s/he is supposed to make with positive probability.

- Assume that the column player chooses H w.p. q and T w.p. $1-q$.
- If the row player chooses H s/he expects to get $q \cdot 1 + (1-q) \cdot (-1)$.
- If the row player chooses T s/he expects to get $q \cdot (-1) + (1-q) \cdot 1$.
- The row player is indifferent between the choices only if

$$q \cdot 1 + (1-q) \cdot (-1) = q \cdot (-1) + (1-q) \cdot 1$$

- From this one can solve $q = \frac{1}{2}$.

- Suppose that you propose to your friend, the row player, that instead of playing a matching the pennies version

	H	T
H	$2,-2$	$-2,2$
T	$-2,2$	$2,-2$

you should play

	H	T
H	$3,-3$	$-2,2$
T	$-2,2$	$1,-1$

- Denote the equilibrium strategy of you, the column player, by $(q, 1-q)$ and that of your friend by $(p, 1-p)$.

- If you choose H you expect $p \cdot (-3) + (1-p) \cdot 2$.
- If you choose T you expect $p \cdot 2 + (1-p) \cdot (-1)$.
- These are equal if $p = \frac{3}{8}$.
- If your friend chooses H s/he expects $q \cdot 3 + (1-q) \cdot (-2)$.
- If your friend chooses T s/he expects $q \cdot (-2) + (1-q) \cdot 1$.
- These are equal if $q = \frac{3}{8}$.

- Notice that your pay-off is given by $\frac{3}{8} \cdot (-3) + \frac{5}{8} \cdot 2 = \frac{1}{8}$.
- So, if the pay-off denotes hundreds of euros you should expect a profit of 1250 in hundred rounds.
- Let us then consider the following game which point to a most important feature of mixed equilibrium strategies

	L	R
L	$2,5$	$4,4+x$
R	$4,2$	$3,4+x$

- Denote the row player's strategy by $(p, 1-p)$ and the column player's strategy by $(q, 1-q)$.

- If the row player chooses L s/he gets $q \cdot 2 + (1 - q) \cdot 4$.
- If the row player chooses R s/he gets $q \cdot 4 + (1 - q) \cdot 3$.
- In equilibrium these are equal or $q = \frac{1}{3}$.
- If the column player chooses L s/he gets $p \cdot 5 + (1 - p) \cdot 2$.
- If the column player chooses R s/he gets $p \cdot (4 + x) + (1 - p) \cdot (4 + x)$.
- In equilibrium these are equal or $p = \frac{2+x}{3}$.

- Here you can see one of the main features of a mixed strategy equilibrium.
- If $x = 0$ one can see clearly what the equilibrium is.
- If x is small and strictly positive one would expect that P2 would put more probability on strategy R.
- But only P1's strategy depends on x.
- The purpose of a player's mixed strategy is to keep the opponent indifferent between his/her choices.
- You can see this as P2's strategy is determined by P1's indifference conditions.

- Let us consider an example that generates a wage distribution.
- There is a large number E of employers with one vacancy, and a large U number of unemployed.
- The unemployed contact the employers in a random manner.
- A pair produces output of worth unity.
- Let the tightness of the market be $\theta \equiv \frac{U}{E}$.
- The probability that an employer is contacted by k unemployed is $e^{-\theta} \frac{\theta^k}{k!}$.
- Assume that the unemployed make an offer to the employer who accepts the lowest one.
- Assume that the unemployed do not know how many other unemployed contact the same employer.

- It is clear that the unemployed must use mixed strategies.
- Denote the mixed strategy by F with support $[l, L]$.
- It is clear that $L = 1$.
- An unemployed who asks wage unity gets it with probability $e^{-\theta}$.
- An unemployed who asks wage l gets a job for certain, and consequently $l = e^{-\theta}$.
- Any wage proposal $\rho \in [l, 1]$ must yield the same utility, i.e., $e^{-\theta}$.
- An unemployed asking ρ expects

$$\rho \sum_{h=0}^{\infty} e^{-\theta} \frac{\theta^h}{h!} (1 - F(\rho))^h = \rho e^{-\theta} e^{\theta(1-F(\rho))} = \rho e^{-\theta F(\rho)}$$

- This must equal $e^{-\theta}$.
- From this equation we can solve

$$F(\rho) = \frac{\theta + log \rho}{\theta}$$