A standard model of bargaining

- Rubinstein's (1982) alternating offers model of bargaining has become a kind of standard as to cake sharing problems.
- There is a cake of unit size that is to be divided between two players.
- The players make offers in an alternating order.
- An offer is implemented if the responder accepts it.
- If not, then the game moves to the next period and the rejector makes an offer.
- Waiting is costly in the sense that the players discount future by factor $0 < \delta < 1$.

- Assume that there are T periods where T is an even number; the first period is period-1.
- Period with name t denotes the time between time instances $t-1$ and t.
- Player1 makes offers in odd periods and Player2 in even periods.
- In period t denote P1's offer by $(x_t, 1-x_t)$ and P2's offer by $(y_t, 1-y_t)$ where the first co-ordinate is P1's share.
- Let us solve the game using backward induction.

- In the last period, T, P2 makes an offer.
- The subgame perfect offer is a division $(y_T, 1 - y_T) = (0, 1)$.
- In the penultimate period, $T - 1$, P1 knows what will happen in period T.
- Thus, any offer that gives P2 less than δ is rejected.
- The subgame perfect offer is a division $(x_{T-1}, 1 - x_{T-1}) = (1 - \delta, \delta)$.

- In period $T-2$ P2 can foresee everything that is going to happen and s/he knows that anything less than $\delta(1-\delta)$ to P1 will be rejected.
- The subgame perfect offer is a division $(y_{T-2}, 1-y_{T-2}) = (\delta(1-\delta), 1-\delta(1-\delta))$.
- By similar logic, the subgame perfect division in period $T-3$ is $(x_{T-3}, 1-x_{T-3}) = (1-\delta(1-\delta(1-\delta)), \delta(1-\delta(1-\delta)))$
- Let us try to see the general structure of the offeres.
- Assume that $T=4$.
- Then the first period offer is$(x_{T-3}, 1-x_{T-3}) = \left(1-\delta+\delta^2-\delta^3, \delta-\delta^2+\delta^3\right)$.

- Make a guess: In general $(x_1, 1-x_1) = \left(1-\delta+\delta^2-...-\delta^{T-1}, \delta-\delta^2+...+\delta^{T-1}\right) = \left(\frac{1-(-\delta)^T}{1-(-\delta)}, \delta\frac{1-(-\delta)^{T-1}}{1-(\delta)}\right) = \left(\frac{1-\delta^T}{1+\delta}, \delta\frac{1+\delta^{T-1}}{1+\delta}\right)$.
- What is remarkable is that in an infinitely long game there is a unique subgame perfect equilibrium in which agreement takes place immediately.
- Further, the division is got by letting T approach infinity $(x_1, 1-x_1) = \left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$.

Learning from others

- Assume that there are two equally likely states of the world G and B.
- Investing a unit in G returns 2, while in B it returns zero.
- People do not know the state but they get a signal g or b.
- $Pr(g|G) = Pr(b|B) > \frac{1}{2}$; $Pr(g|B) = Pr(b|G) < \frac{1}{2}$.
- People make their investment decisions in a sequence, and observe the precedessors' actions but not their signals.
- Based on this they update their beliefs.
- Assume that people who are indifferent between investing and not do as their precedessor did.

- Assume that the true state (unknown to everyone) is B, and that the first person gets signal g.

- Then s/he invests.

- The second person observes this and can infer the first person's signal.

- If the second person gets signal b s/he knows two opposing signals, and has beliefs that both states are equally likely.

- But s/he invests as the precedessor did so.

- Consider the third person and assume that s/he gets signal b.
- Now there are altogether two correct signals b, and one incorrect signal g.
- The third person regards as possible strings of signals (g, b, b) and (g, g, b).
- One can update his/her beliefs by the Bayes's rule or one can observe that the middle signals cancel out as does his/her and the first person's signals.
- Consequently, s/he is indifferent between investing and not.
- As person-2 invested s/he also invests.
- This means that regardless of his/her signal the fourth person also invests as do all the succeeding persons.
- Now there is a herd where everyone invests even though the state of the world is bad.