## Course syllabus and schedule

(1) (MM) T. Schelling: Micromotives and Macrobehavior, 1978, W. W. Norton and Company.
(2) (SC) T. Schelling: The Strategy of Conflict, 1980, Harvard University, Cambridge, Massachusetts.

These are required reading and the exam is based on both books and the lectures.

- Lecture -1. Some basics of game theory: Normal form games.
- Lecture 0. Some basics of game theory: Sequential form games.
- Lecture 1. Chapters 1 in MM.
- Lecture 2. Chapters 2-3 in MM.
- Lecture 3. Chapter 4 in MM.
- Lecture 4. Chapter 5 in MM.
- Lecture 5. Chapters 6-7 in MM.
- Lecture 6. Chapter 1 in SC.
- Lecture 7. Chapter 2 in SC.
- Lecture 8. Chapter 3 in SC.
- Lecture 9. Chapter 4 in SC.
- Lecture 10. Modern formulations of some of the preceding ideas.


## Some basic game theory

- In the first two lectures we go through some basics of game theory.
- You are supposed to ponder upon the writings and ideas of Schelling using this machinery.
- This material corresponds to modern text books in game theory.
- Many of the Schelling's ways of expressing ideas or analysing strategic situations are a little old fashioned; this should not present problems.


## Normal form games

- This is a way of depicting games where the order of making decisions is not important.
- The interpretation is that players make their choices simultaneously, or that they do not know what others have chosen at the time of their own decisions.
- We consider mostly two-player games; the extension to $n$-player games is straightforward.
- Players are called $P 1$ and $P 2$.
- They have action/choice/strategy sets $A 1$ and $A 2$.
- If the players' choices are $a_{1}$ and $a_{2}$ their utilities are

$$
u_{1}\left(a_{1}, a_{2}\right)
$$

and

$$
u_{2}\left(a_{2}, a_{1}\right)
$$

- The players' objective is to attain as high level of utility as possible

$$
\max _{a \in A_{1}} u_{1}\left(a, a_{2}\right)
$$

- Notice that for some reason in the maximisation it is assumed that $P 1$ knows, or expects, $P 2$ to choose action $a_{2}$.
- If $P 1$ does not know what $P 2$ is going to choose then $\mathrm{s} / \mathrm{he}$ must have expectation about $P 2$ 's choices (probability distribution over A2).
- Otherwise one cannot formulate the problem of the players.
- This is solved by the solution concept of Nash-equilibrium.


## Definition

A Nash-equilibrium is a pair of choices $\left(a_{1}, a_{2}\right) \in A 1 \times A 2$ such that $a_{1}$ is a solution to $\max _{a \in A 1} u_{1}\left(a, a_{2}\right)$ and $a_{2}$ is a solution to $\max _{a \in A 2} u_{2}\left(a_{1}, a\right)$.

## Prisoners' dilemma

$$
\begin{array}{ccc} 
& c & d \\
c & 2,2 & 0,3 \\
d & 3,0 & 1,1
\end{array}
$$

- Here one choice, $d$, is a dominating one, and the Nash-equilibrium is $(d, d)$.


## Battle of the sexes

$$
\begin{array}{ccc} 
& \text { bo } & \text { ba } \\
\text { bo } & 2,1 & 0,0 \\
\text { ba } & 0,0 & 1,2
\end{array}
$$

- Here the man wants to go to a boxing match, and the woman to the ballet; however, the most important thing is that both get to go together to a same place.
- There are two Nash-equilibria.


## Co-ordination game

$$
\begin{array}{ccc} 
& I & r \\
l & 9,9 & 0,0 \\
r & 0,0 & 1,1
\end{array}
$$

- The most important thing is to make the same choice as the other player.
- There are two Nash-equilibria.


## Congestion game

$$
\begin{array}{ccc} 
& l & r \\
l & -1,-1 & 1,1 \\
r & 1,1 & -1,-1
\end{array}
$$

- The most important thing is to make a choice different from the other player's choice.
- There are two Nash-equilibria.


## Matching pennies

$$
\begin{array}{ccc} 
& h & t \\
h & -1,1 & 1,-1 \\
t & 1,-1 & -1,1
\end{array}
$$

- Two players choose simultaneously either heads or tails.
- If both choose the same player1 loses one euro and player2 gets one euro.
- If they choose differently then payoffs go the other way.
- There is no Nash-equilibrium in this game.
- However, there is so called mixed strategy Nash-equilibrium.
- We shall go into this in more detail later.
- Let us think about some examples.
- Two persons arrive simultaneously at a very narrow doorway from opposite directions. Who goes first?
- Two persons simultaneously and secretly write a number on a paper.
- Then they reveal the numbers and the person with the highest number wins.
- What is the Nash-equilibrium?
- The governments of two neighbouring countries decide on tax policy.
- Both would like a high tax rate if the other chooses a high tax rate because this way they can steal the most from the people.
- But if one chooses a high tax rate and the other a low, then the latter can attract business from the former country which then suffers.
- Two ladies ponder about getting a really fancy dress for the independence day party.
- There is a particular specimen of which two units are available.
- If both ladies happen to choose the same dress it will be an embarrasment, and on top of that, because it is such a wonderful piece of clothing, it costs a lot.
- If a lady chooses a lesser dress it is certain that she will not attract any attention, perhaps does not even get her picture in the newspapers.


## Equilibrium is about expectations

- Consider the battle of the sexes game.
- If the row player, man, expects the column player to choose bo he chooses bo.
- If the column player, woman, expects the row player to choose ba she chooses ba.
- Both make the maximising choice given their expectations.
- But the outcome does not qualify as Nash-equilibrium.
- For the Nash-equilibrium it is not enough to maximise but to have correct expectations, too.


## Rationality

- We assume that the players are rational.
- This means that they make optimal choices given their information/expectations.
- As demonstrated above, Nash-equilibrium requires more, namely, correct expectations.
- How do the players come to the correct expectations?
- The concept of Nash-equilibrium is silent about this.
- There are many stories about how to achieve the equilibrium but at the present we do not say more about this.

