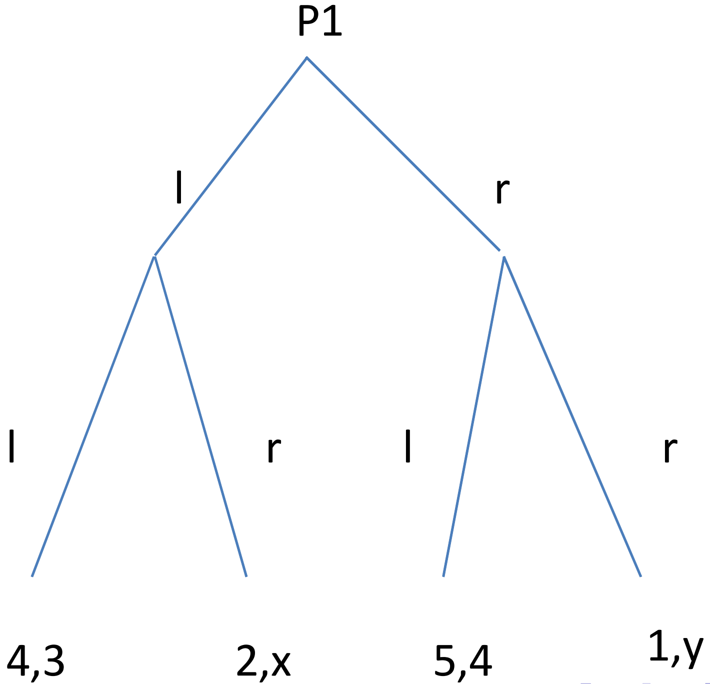
Extensive form games

- This manner of depicting games is particularly suitable to situations where the players make their choices sequentially.
- One uses game trees where there is an initial node from which subsequent nodes can be reached by edges that connect nodes.
- At each node some player makes a choice.
- Because one has to keep track of the order of the moves the formal presentation of the game and especially the strategies/actions is more complicated than for the normal form games.
- The formal definition of an extensive form game is quite complicated.

- Here we focus on strategies and assume that on informal level a game tree is almost self-explanatory.


P1
l
r
l
r
l
r
4,3
2,x
5,4
1,y

- In the game tree there are two players.
- Player1 makes his/her choice first.
- Player2 observes the choice and makes his/her choice then.

- Both players' action set is $A = \{l, r\}$, or consists of 'left' and 'right'.
- However, player2's strategy set is much more complicated.
- Because s/he can condition his/her choice on what player1 does his/her possible strategies are given by

$$S2 = \{(l,l),(l,r),(r,l),(r,r)\}$$

- The first co-ordinate in any strategy tells what P2 does when P1 has chosen l, and the second co-ordinate what to do when P1 has chosen r.

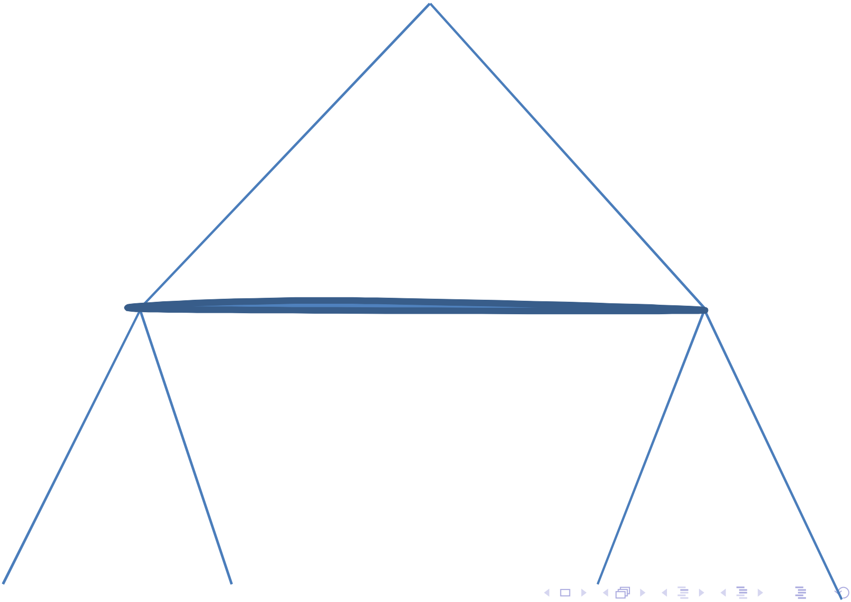
- Let us study under which conditions various strategies constitute a Nash-equilibrium.
- 1. $(l,(l,l))$: P1 gets 4 and P2 gets 3. First, it is clear that $x \leq 3$ must hold. Had P1 chosen r s/he would have got 5. Consequently, this is not a Nash-equilibrium.
- 2. $(l,(l,r))$: P1 gets 2 and P2 gets 3. Again $x \leq 3$ must hold. If P1 had chosen r s/he would have got 1. So, this is a Nash-equilibrium.
- 3. $(l,(r,l))$: P1 gets 2 and P2 gets x. The choice of P2 is optimal if $x \geq 3$. Had P1 chosen r s/he would have got 5 which more than 2. Not a Nash-equilibrium.
- 4. $(l,(r,r))$: P1 gets 2 and P2 gets x. The choice of P2 is optimal if $x \geq 3$. Had P1 chosen r s/he would have got 1. So, this is a Nash-equilibrium.

- 5. $(r,(l,l))$: P1 gets 5 and P2 gets 4. The choice of P2 is optimal if $y \leq 4$. Had P1 chosen l s/he would have got 4 which is less than 5. This is a Nash-equilibrium.
- 6. $(r,(l,r))$: P1 gets 1 and P2 gets y. The choice of P2 is optimal if $y \geq 4$. Had P1 chosen l s/he would have got 4 which is more than 1. Not a Nash-equilibrium.
- 7. $(r,(r,l))$: P1 gets 5 and P2 gets 4. The choice of P2 is optimal if $y \leq 4$. Had P1 chosen l s/he would have got 2 which is less than 5. This is a Nash-equilibrium.
- 8. $(r,(r,r))$: P1 gets 1 and P2 gets y. The choice of P2 is optimal if $y \geq 4$. Had P1 chosen l s/he would have got 2 which is more than 1. Not a Nash-equilibrium.

- Assume that $y = 1$ and $x = 9$.
- Consider equilibrium $(l, (r, r))$.
- This is problematic.
- An interpretation of this equilibrium is that P2 threatens P1 that if the latter chooses r P2 will choose r.
- For this reason P1 actually chooses l.
- But this threat is empty as a player in a node following P1's choice of r makes a decision between getting 1 and 4.
- A rational player will choose 4, or in this case l.
- In game theoretic parlance this is not a subgame perfect equilibrium.

- Rosenthal's centipede game is a striking example where requirement of subgame perfectness leads
- http://www.econport.org/econport/request?page=man_gametheory_
- It is a good idea to solve extensive form games from the end to the beginning.
- Figuring out at each node the optimal decision one comes up with a subgame perfect equilibrium.
- The procedure is called backward induction.

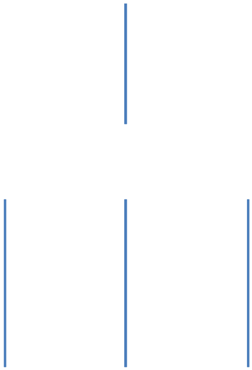
- Simultaneous moves can be modelled by combining nodes into sets of nodes called information sets.



Example Marienbad-game

- From Ritzberger, Foundations of non-cooperative game theory.
- There are two players and m^2 matches in a pyramid shape such that in the first row there is one match, in the second row there are three matches, and in the m^{th} row there are $2m-1$ matches.
- First player removes any number $k \geq 1$ matches from exactly one row.
- Then the other player does analogously, and the players alternate turns until one of the players removes the last match(es).
- S/he loses.

- The $m = 2$ game is depicted in the figure



- and its extensive form is drawn on the white board

- In the extensive form the game tree ends in positions where it is clear who is the winner.
- It is immeadiate that P1 wins by removing all the sticks in the second row, that is three sticks.
- Try to draw the game tree for $m = 3$ game.
- Try to figure out whether it is obvious in complete information games what is the equilibrium.