Sorting along age and income

- Focus on continuous variables.
- Consider a large population of people uniformly distributed between ages 0 and 100.
- There are two rooms where the people can go.
- They prefer to go to the room where the average age is closest to his/her own.
- Equilibrium is pretty simple: everyone less than 50 goes to one room and everyone over 50 to the other room.

- What if everyone wants to be in the room where the age in the pth percentile is closest to his/her own age.
- Assume that in one room there are people aged between 0 and x, and that the rest are in the other room.
- The pth percentile in the first room is px, and in the other room $x + p(100 - x)$.
- Person x has to be indifferent between the two rooms, or

$$x - px = x + p(100 - x) - x$$

which yields $x = p100$.

- The interesting cases are such that people have a preference about a population statistic to which they contribute, and where there is positive correlation between the preference and the contribution.
- Consider the example in MM where there are two rooms and people prefer the room the closer to 55% of the population there is.
- Given any percentages $x\%$ and $100\% - x\%$ where $x\% < 50\%$, it is the case that 50% is equally close to $x\%$ and $100\% - x\%$.
- But then necessarily 55% is further away from $x\%$ than $100\% - x\%$.
- Consequently people always want to go to the room with $100\% - x\%$ of the people.
- Notice that 100% is further away from 55% than 15%.
- Thus, one could improve everyone's situation by moving 15% of the people into the other room.

- More interesting application of a continuous variable is the following.

Example

Statistical discrimination.
Assume that there are two types of people; men and women, or blacks and whites, or natives and immigrants.
Call the types a and b.
Each type's productivity is similarly distributed on the unit interval, say it is uniform on $[0, 1]$.
When looking for a job a person is interviewed.
Assume that if an interviewer is of the same type as the job applicant s/he can infer the productivity accurately.
If an interviewer is of the other type s/he gets an informative but imperfect signal of the productivity.

Example

(continued). Assume that given productivity $z \in [0,1]$ the interviewer gets signal that is distributed by density

$$f(s|z) = \begin{cases} \delta_\alpha, \textit{ if} & s = z \\ 1-\alpha, x > z & s \neq z \end{cases}$$

where δ_α is Dirac-measure or point probability.
If the interviewer gets signal s s/he calculates that the probability of the job applicant being of type x is

$$f(x|s) = \frac{f(s|x)f(x)}{\int_0^1 f(s|x)f(x)dx}$$

which is just the Bayes's theorem.

Example

(continued). If $x = s$ then the above formula gives $\frac{\delta_\alpha}{\delta_\alpha + \int_0^1 f(s|x)(1-\alpha)dx} = \delta_\alpha$.
If $x \neq s$ we get density $1-\alpha$.
Given that the interviewer observes s s/he calculates the expected productivity of the job applicant

$$\alpha s + (1-\alpha)\int_0^1 x dx = \alpha s + \frac{1}{2}(1-\alpha)$$

Example

(continued). A job applicant who knows his/her productivity x rationally expects the interviewer to evaluate him/her as

$$\int_0^1 \left(\alpha s + \frac{1}{2}(1-\alpha)\right)(1-\alpha)ds + \alpha\left(\alpha x + \frac{1}{2}(1-\alpha)\right)$$

which equals

$$\alpha(1-\alpha) + \frac{1}{2}(1-\alpha)^2 + \alpha^2 x$$

One easily finds that this is less than x if $x > \frac{1}{2}$ and it is more than x if $x < \frac{1}{2}$.

Example

(continued). Assume that $\alpha = \frac{4}{10}$.
Assume that all the interviewers are of type a.
This is unfortunate for the aggregate success of type b job applicants.
Assume that the interviewer interviews five applicants of both type.

Example

(continued). Assume that the applicants of type a are of productivities $\frac{2}{10}$, $\frac{4}{10}$, $\frac{5}{10}$, $\frac{6}{10}$ and $\frac{8}{10}$.
Assume that the applicants of type b are of productivities $\frac{3}{10}$, $\frac{5}{10}$, $\frac{6}{10}$, $\frac{8}{10}$ and $\frac{9}{10}$.
Type a applicants' productivities are perfectly observed.
Type b applicants' productivities are estimated at $\frac{468}{1000}$, $\frac{500}{1000}$, $\frac{516}{1000}$, $\frac{548}{1000}$ and $\frac{564}{1000}$.

Example

(continued). But now two of the type a applicants are ranked higher than the best of the type b applicants.
If there is only one or two vacancies only type a applicants are hired.

- The rest of MM continues along the familiar lines and nothing particularly new emerges.
- Two central points are i) that there are a lot of interesting situations where strategic behaviour may have bad consequences.
- There is not, however, a single model that can be used to analyse them.
- And ii) strategic behaviour leads to inefficient outcomes.

- Let us see how obvious the second point is when the pay-offs are randomly generated.
- Use this link http://www.random.org/integers/