Focal points

- In SS Schelling studies the problems of 'selecting' one of the equilibria when there are many.
- The basic idea is that often times there are some natural candidates onto which the players can co-ordinate.
- Why they are natural is more difficult to formalise.
- Perhaps some common culture or unexplained attraction of symmetry are the determinants.
- For focal points to work there must be a signal perceived by all players.
- The signal must be unambiguous.
- Further, each player must know that others have perceived it, and each player must know that this is the case etc.

- Schelling studies these things in a setting he calls tacit bargaining.
- Players have to make a choice without communicating with each other.
- The examples he goes through do not necessarily deal with bargianing/cake sharing.
- Let us cover some of them here.

Common interest

1. You and your friend must write down 'heads' or 'tails' simultaneously. If you write the same thing both of you win some money; otherwise you do not win.
2. Pick one of the numbers 7, 100, 13, 261, 99 and 555. If everyone picks the same number you win some money; otherwise you do not win.
3. Pick one of the numbers $\sqrt{2}$, π, 13, 19, 91 and 92. If everyone picks the same number you win some money; otherwise you do not win.

4. You are supposed to meet your friend in Helsinki at 11.59 tomorrow. Where would you go to meet him/her if the only thing you know is the time, and that your friend is in the same situation. If you are a foreigner assume that your friend is, too. If you are a native assume that your friend is, too.
5. You are supposed to meet your friend in Helsinki at 11.59 tomorrow. Where would you go to meet him/her if the only thing you know is the time, and that your friend is in the same situation. If you are a foreigner assume that your friend is a native Finn. If you are a native assume that your friend is a foreigner.

6. You and another person must write down 'heads' or 'tails' simultaneously. If both of you write 'heads' you get 3 and the other person 2. If both of you write 'tails' you get 2 and the other person 3. Otherwise neither of you gets anything.
7. You are given a piece of paper with X written on it. Another person is given a blank piece of paper. You can either erase the X or leave it there. The other person can either write X on his/her piece of paper or leave it blank. In the end the pieces are collected and if both feature X you get 2 and the other person gets 3. If both are blank you get 3 and the other person 2. Otherwise neither of you gets anything.
8. You and another person are given 100 to divide. You must write down how much of it you want as must the other person. If the sum turns out at most 100 you get what you wrote down. If the sum exceeds 100 neither of you gets anything.

Electronic mail game

Example

For co-ordination to take place it is necessary that everyone knows where to co-ordinate, and everyone knows that everyone knows it, and so on.
Consider two fighter pilots who must co-ordinate to a target.
Pilot A chooses the target and transmits the information to pilot B.
The transmission fails with probability p.
If pilot A proceeds to attack s/he has not protection with probability p, and the attack fails.
With probability $1 - p$ it succeeds, provided that pilot B is willing to attack on having received the message.

Example

(Continued) But pilot B may have doubts about pilot A's willingness to attack unless s/he knows that pilot B has received the message.
Pilot B consequently transmits to pilot A that s/he received the message.
But now pilot B cannot be certain whether pilot A has received pilot B's message; with probability $1-p$ s/he has but with probability p s/he has not.
If pilot B's message is successful both know where to attack, and both know that the other one knows where to attack.
But even in the case of a successful reply message pilot B does not know that pilot A knows that pilot B knows where to attack.

Example

(continued) This is because pilot B does not know whether his/her reply message was successful.
So s/he regards it possible that it did not go through.
But then pilot A regards it as possible that his/her original message did not go through.
What if pilot A attacks even if s/he does not receive the reply message.
That would be stupid because the probability of the original message being lost is p while the probability of the reply message being lost is $(1-p)p$.
Thus, the probability of unco-ordinated attack would be greater than that of the co-ordinated attack (assuming pilot B attacks).

Example

(continued). If pilot A attacks once s/he gets the reply message, and pilot B if s/he gets the first message the probability of a co-ordinated attack is $(1-p)^2$ and that of unco-ordinated attack $(1-p)p$.
Now it is pilot B who attacks alone.
There is no improvement compared to the case where the pilot have agreed to send just one message.
There does not exist any plan that improves the situation.

Example

(continued). For instance a plan where both attack if they have received k messages of scheduled $k+l$ messages necessarily fails.
If pilot B gets the kth message but not the $(k+1)$st s/he reasons that it is more likely that his/her kth message lost than the reply of A lost.
But then B does not attack.
Thus, there is co-ordinated attack only if all the scheduled messages get through.
Otherwise there is no attack or just pilot B attacks if all but the last scheduled message gets through.

Example

Consider the following game(1)

	$t1$	$t2$	$t3$	$t4$
$s1$	$4,3$	$4,7$	$1,4$	$10,2$
$s2$	$5,5$	$5,1$	$0,0$	$3,4$
$s3$	$3,9$	$3,3$	$-1,6$	$4,8$

There is one Nash-equilibrium in pure strategies $(s2, t1)$.
Let us consider what kind of information the players must possess to end up in the Nash-equilibrium.
Assume that they are rational and know the pay-offs but that there is uncertainty about the common knowledge of rationality.

Example

(continued). Let K denote the knowledge operator.
Let R denote the property of being rational.
We denote the fact that player 2 is rational by $P2R$.
Using these symbols we can say 'player 1 knows that player 2 is rational' by $P1KP2\, is\, rational$.
Round 0. Both P1 and P2 are rational. Neither knows anything about the other one's rationality.
Consequently, each thinks that the opponent can do anything.

Example

(continued). For P1 action $s1$ dominates $s3$. Thus s/he never chooses $s3$.
For P2 $t1$ dominates $t4$.
P1 thinks that s/he plays game(2)

	$t1$	$t2$	$t3$	$t4$
$s1$	$4,3$	$4,7$	$1,4$	$10,2$
$s2$	$5,5$	$5,1$	$0,0$	$3,4$

Example

(continued). P2 thinks that s/he plays game(3)

	$t1$	$t2$	$t3$
$s1$	$4,3$	$4,7$	$1,4$
$s2$	$5,5$	$5,1$	$0,0$
$s3$	$3,9$	$3,3$	$-1,6$

Example

(continued). Round 1. Now we assume that $P1KP2R$ and $P2KP1R$.

Thus, both players know the opponent's reasoning in round 0.

Thus, both consider the game(4)

	$t1$	$t2$	$t3$
$s1$	$4,3$	$4,7$	$1,4$
$s2$	$5,5$	$5,1$	$0,0$

Example

(continued). Since there is just one round of knowledge about rationality it is the case that $P1 \sim KP2KP1R$.
Thus, P1 cannot make the inference that P2 made above, namely, that game(4) is played.
P1's expectation about the game P2 is playing is based on the reasoning P2 made in round 0 that game(3) is played.
Similarly, P2's expectation is about P1 is based on P1's round-0 reasoning, or that game(2) is played.
Now none of P1's actions are dominated and s/he plays game(4).

Example

(continued). P2 expects P1 not to use action $s3$.
Thus, his/her rational response is to play game(5)

	$t1$	$t2$
$s1$	$4,3$	$4,7$
$s2$	$5,5$	$5,1$

since $t3$ is dominated by $t2$.
You can test your own reasoning to complete rounds 2 and 3 after which the Nash-equilibrium is reached.
You should note that at this point the players do not know it but round 4 is required.