About communicating information credibly

- In sections 4 and 5 Schelling discusses how to improve game theory and its limitations.
- Many of the 'defects' of game theory he writes about have been taken care of in the subsequent literature.
- Schelling touches many things of which this lecture focuses on how one can communicate private information.

Example

Consider potential workers who can be of high productivity or low productivity.
Assume that $1/3$ of each cohort of people are of high productivity and the rest of low productivity.
Workers know their productivity but it is their private information.
Before entering the job market they can acquire education.

Example

(continued) Getting education is costly.
Level e of education costs $c(e) = e^2$ for the high types and $d(e) = 2e^2$ for the low types.
The productivity of high types is A and that of low types is $\frac{1}{2}A$ where $A > 0$.
The workers are paid exactly their expected productivity in a competitive labour market.

Example

(continued). Let us construct an equilibrium where no-one acquires education.
Then employers cannot separate the workers according to their productivity.
An employer is worth $\frac{1}{3}A + \frac{2}{3}\frac{1}{2}A = \frac{2}{3}A$ which is his/her payoff.
The equilibrium is supported by beliefs that anyone who acquires education is of low productivity.

Example

(continued) Let us construct an equilibrium where high productivity types acquire level e of education but low productivity types do not. High productivity worker gets pay-off

$$A - e^2$$

and low productivity worker

$$\frac{1}{2}A$$

Example

(continued) For this to be an equilibrium no worker should benefit from pretending to be of the other type, i.e.,

$$A - e^2 \geq \frac{1}{2}A$$

$$\frac{1}{2}A \geq A - 2e^2$$

First of the expressions is equivalent to

$$\sqrt{\frac{1}{2}A} \geq e$$

and the second is equivalent to

$$e \geq \sqrt{\frac{1}{4}A}$$

Example

(continued) The equilibrium is supported by beliefs that postulate that anyone acquiring anything but the equilibrium level of education is of low type.

Notice that education is pure waste; it does not increase productivity.

In the least cost separating equilibrium $e = \sqrt{\frac{1}{4}A}$.

Costless communication or signalling

Example

There are two players, a sender and a receiver.
The sender observes a state $t \in [0, 1]$ that is his/her private knowledge.
The state is uniformly distributed.
S/he sends a message $m \in [0, 1]$ to the receiver.
Then the receiver takes an action $y \in [0, 1]$.

Example

(continued) The sender's pay-off is $-(y-(t+b))^2$.
The receiver's pay-off is $-(y-t)^2$.
Denote the sender's strategy by $s(t)$.
Assume that in equilibrium $s(t) = t$.
Then the receiver always chooses $y = t$ and gets zero.
The sender gets $-b^2$.
The sender has a profitable deviation $m = t + b$ which yields pay-off zero.
Thus, truthful communication is not possible in equilibrium.

Example

(continued) What if the sender always sends the same signal $s(t) = a \in [0, 1]$.
Then no information is transmitted and the receiver maximises his/her expected welfare by $y = \frac{1}{2}$.

Example

(continued) Let us construct an equilibrium in which some information is transmitted.
If $t \in [0, t_1]$ then the sender sends message m_1 and otherwise message m_2.
When the receiver observes m_1 s/he knows that the true state is uniformly distributed on $[0, t_1]$.
His/her optimal action is $y = \frac{1}{2} t_1$.
Analogously, if s/he observes m_2 his/her optimal action is $y = \frac{1}{2}(t_1 + 1)$.

Example

(continued) If s/he observes anything else s/he has either of the above beliefs (it does not matter which).
If the sender changes his/her strategy s/he can change the receiver's action from $\frac{1}{2}(t_1+1)$ to $\frac{1}{2}t_1$ or vice versa.
At $t = t_1$ s/he must be indifferent between the actions

$$t_1 + b = \frac{1}{2}\left[\frac{1}{2}t_1 + \frac{1}{2}(t_1+1)\right]$$

from which we can solve

$$t_1 = \frac{1}{2} - 2b$$

This makes sense only if $b < \frac{1}{4}$.
The above is a necessary condition for equilibrium.