Game theory lecture 3

September 16, 2012

Non-cooperative and cooperative games

- In non-co-operative games the focus is on individual players and their actions.
- In co-operative games one does not consider what individual players do; quite to the contrary the main role is played by a function that tells the worth of each coalition.
- The focus of this course is non-co-operative games.

Classification of games

Normal form games and extensive form games

- This is a classication of non-co-operative games.
- In normal form games the players can be thought to choose their actions simultaneously.
- In extensive form games the sequential structure of the strategic situations is important.
- We start with normal form games.
- Sometimes a distinction between static games and dynamic games is made when referring to the normal form games and extensive form games, but it is slightly misleading.

Classification of games

Games with complete and incomplete information

- This classication pertains to non-cooperative games.
- It makes a huge difference whether there is complete information or whether the players do not know some relevant aspects of their opponents.
- Games of incomplete information are considered in the latter part of the course.

Examples of strategic situations

Example1.

- You are buying a second-hand bicycle.
- You value it at 100 euros.
- The seller values it at 50 euros.
- You offer 50 euros.
- The seller asks 100 euros.
- How does the situation end?
- You would like to commit not to offer anything more than 50 euros. How can you commit?

Example2.

- It used to be the case in New York (and might still be) that a new owner of a rent-controlled apartment house could evict one of the tenants so as to live in the building him/herself.
- But this right actually turned into a right to evict everybody.
- The new owner can offer the first tenant a choice between being evicted or leaving voluntarily and getting 100 dollars as a go-away gift.
- The s/he can make the same offer to the next tenant.

Example3.

- Bicycles can be good or bad.
- The owners know the condition of their bikes.
- A good bike is worth 100 euros and a bad bike worth 15 euros.
- A person comes to you and offers to sell his/her bike for 90 euros.
- What do you infer about the conditions of the bike?

Example4.

- The Finnish government aims to raise the taxation of entrepreneurs.
- The entrepreneurs claim/threaten to move to Estonia (Kauppalehti September the 8th 2011).
- Is the threat credible?
- Is moving to Estonia the optimal response?

Example5.

- There are 100 parking lots in the city centre.
- Each morning 105 drivers come/would like to come to the centre around 8 o'clock for work.
- What is going to happen?

Example6.

- On Thursday the residents of Santa Fe want to go to El Faro bar.
- But they only want to go there if less than 60% of them show up.
- If more than 60% show up the bar is too crowded and everyone would rather be at home watching tv and drinking beer.
- If everyone uses the same strategy everyone ends up doing what they would not like to do.

Example7.

- After a long construction work there are two routes available to the residents of A to go to B.
- Both routes are equally good and the residents want to choose the route chosen by the minority to avoid congestion.
- What happens?

Examples of strategic situations

Example8.

- Citizens may vote for one of the two presidential candidates.
- Each citizen has his or her favourite whom s/he would like to see elected.
- Voting is costly, though; one has to go some distance to vote and there is all kinds of hassle to be expected.
- Besides if everyone else votes the chance that one's vote has any effect is practically zero.
- Why do people vote?

- Let us try to model and solve some of the above problems later on once we have learned some techniques.
- Next we go through some standard examples of games.
- Most of them can be found in the text book.
- Notice that Osborne does not assume von Neumann-Morgenstern preferences or preferences of expected utility form until he studies mixed strategies on page 102.
- The reader may not notice this fact; it does not matter much since Osborne only deals with pure strategies in the beginning of the book.
- I, however, assume that the preferences are of von Neumann-Morgenstern type from the outset.

Examples of normal form games

Example1. Prisoners' dilemma

	C	D
C	$2, 2$	$0, 3$
D	$3, 0$	$1, 1$

- Player1 chooses the row and player2 the column.
- The first number in each sell denotes player1's pay-off, and the second number that of player2.
- The players make their choices independently without knowing what the other player chooses.

Example2. Battle of the sexes

	Bo	*Ba*
Bo	2, 1	0, 0
Ba	0, 0	1, 2

Example3. Stag hunt

	S	H
S	$2,2$	$0,1$
H	$1,0$	$1,1$

Example4. Hawk and dove

	H	D
H	$(v-c)/2, (v-c)/2$	$v, 0$
D	$0, v$	$v/2, v/2$

where we assume that $v > c$.

Examples of normal form games

Example5.

	L	R
U	2, 1	0, 0
M	1, 1	2, 2
D	0, 4	3, 3

Example6.

	L	R
U	1, 1	0, 0
M	1, 1	2, 1
D	0, 0	2, 1

Examples of normal form games

Example7. Co-ordination game

	a	b
a	$9,9$	$0,0$
b	$0,0$	$1,1$

Example8.

	a	b
a	$9,9$	$0,8$
b	$8,0$	$7,7$

Dominance

- Which are plausible outcomes? Why?
- One principle is dominance. There are two types.
- Strict dominance and weak dominance.
- One should expect that a strictly dominated action is never cho sen.
- One could iteratively remove all strictly dominated actions but this does not typically lead to a unique outcome.
- Iteratively moving weakly dominated actions may lead to different outcomes depending on the order of removal.
- The bottom line is that no form of iteratively removing dominated actions/strategies provides a foundation for a solution to games.
- To consider iterative dominance arguments, and to proceed anyway, we first need to carefully formalise what a normal form game is.

Dominance

Normal form representation. A normal form game is given by $\Gamma = \left(N, \{A_i\}_{i \in N}, \{u_i\}_{i \in N}\right)$ where $N = \{1, ..., n\}$is the set of players, A_i is player i's set of actions and u_i is his/her utility function, $u_i : \times_{i \in N} A_i \to \mathbb{R}$.

Strict **dominance.** In a normal form game $\Gamma = \left(N, \{A_i\}_{i \in N}, \{u_i\}_{i \in N}\right)$ action $a_i \in A_i$is strictly dominated if there is a different action $a_i' \in A_i$ such that $u_i(a_i, a_{-i}) < u_i(a_i', a_{-i})$ for all actions of other players $a_{-i} \in \times_{j \neq i} A_j$.

Dominance

Weak dominance. In a normal form game $\Gamma = \left(N, \{A_i\}_{i \in N}, \{u_i\}_{i \in N}\right)$ action $a_i \in A_i$is weakly dominated if there is a different action $a_i' \in A_i$ such that $u_i(a_i, a_{-i}) \leq u_i(a_i', a_{-i})$ for all actions of other players $a_{-i} \in \times_{j \neq i} A_j$, and $u_i(a_i, a_{-i}) < u_i(a_i', a_{-i})$ for at least one profile of actions of other players $a_{-i} \in \times_{j \neq i} A_j$.

Nash equilibrium

- The solution concept that is adopted is (surprise, surprise) that of Nash- equilibrium.

Nash equilibrium. In a normal form game $\Gamma = \left(N, \{A_i\}_{i\in N}, \{u_i\}_{i\in N}\right)$ an action profile $a = (a_1, ..., a_n) \in \times_{i\in N} A_i$ is a Nash equilibrium if $u_i(a_i, a_{-i}) \geq u_i(a_i', a_{-i})$ for all $a_i' \in A_i$ and for all $i \in N$.

- One of the advantages of Nash-equilibrium is that it usually exists in situations of interest.
- One of the disadvantages is that there are typically a multiplicity of them.

Solving games

- Let us consider a couple of cases where one can use dominance.

	a	b	c	d	e
A	$3,1$	$4,4$	$9,0$	$7,4$	$2,5$
B	$10,6$	$3,6$	$9,2$	$8,2$	$3,3$
C	$2,1$	$2,1$	$4,2$	$9,2$	$9,4$
D	$7,2$	$0,5$	$3,8$	$8,8$	$3,9$

- If only strict dominance is allowed we get

	a	b	e
A	$3,1$	$4,4$	$2,5$
B	$10,6$	$3,6$	$3,3$
C	$2,1$	$2,1$	$9,4$
D	$7,2$	$0,5$	$3,9$

- If also weak dominance is allowed we get

	b	e
A	$4,4$	$2,5$
B	$3,6$	$3,3$
C	$2,1$	$9,4$

Solving games

- Guess 2=3 of the average.
- Each of you can choose a number between 0 and 100.
- We calculate the average of that number.
- Then we take 2/3 of the average.
- The person whose guess is closest to this magnitude gets a prize.
- It is clear that all choices that are larger than 67 are strictly dominated.
- Once this is understood it is evident that all choices that are larger than 45 are strictly dominated.
- Going on like this one notes that all choices that are different from zero are strictly dominated.

Solving games

- A curious example about strict dominance.
- Let the set of players be $N = \{1, 2\}$,the action sets $A_1 = A_2 = [0, 1]$and the utility functions $u_i : A_i \times A_j \to \mathbb{R}$

$$u_i(x, y) = x \; if \; x < 1$$

$$u_i(1, y) = 0 \; if \; y < 1$$

$$u_i(1, 1) = 1$$

- Each action except 1 is strictly dominated, and $(1, 1)$ is the unique Nash- equilibrium.
- Eliminating all actions $A_i \setminus \{1, x\}$ $x < 1$, gives the following 2x2 game

	1	x
1	$1, 1$	$0, x$
x	$x, 0$	x, x