# Monetary theory

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- In standard neoclassical models, in particular in general equilibrium theory, there is no need for money.
- The markets are so perfect that only relative relative prices are important and exchange happens without a mediating means of trade.
- Some imperfections about the markets have to be assumed to create a role for money.
- Not too surprisingly search frictions provide a possibility.

- Typically money is assumed to possess at least the following three properties:

1. Unit of account
2. Store of value
3. Medium of exchange

- It is the last property that is going to play the key role in the present modelling excercise.
- Note also that if there is an object that functions as a medium of exchange it necessarily serves as a store of value, too.

- The basic idea is that people have heterogeneous preferences, and they produce different goods.
- If their meetings are stochastic then a seller of bread who meets a lecturer of economics most likely cannot agree on trading.
- This problem is called double-coincidence of wants problem.
- Introduction of primitive form of money eases the problem.
- Assume that there are 100 different goods and equally many preferences for these particular goods.
- Assume that both preferences and production are uniformly distributed in the population.
- Now, when two people meet the probability that both desire what the other has is of order $\frac{1}{10000}$.
- Assume that half of the population goes around with one unit of money that is an intrinsically worthless object.
- In a meeting the probability of trade is of order $\frac{1}{2}\frac{1}{2}\frac{1}{10000} + 2\frac{1}{2}\frac{1}{2}\frac{1}{100}$.

- The following model is from Kiyotaki and Wright (1993, American Economic Review).
- It is very rudimentary and stylised as both goods and money are indivisible.
- The agents can hold at most one unit of money or one unit of a good at a time.
- The economy proceeds in discrete time.
- In any period an agent's preference is randomly determined from a uniform distribution on the unit circle.
- If the agent's preference is for good $i \in [0,1]$ where 0 and 1 indicate the same good, then s/he gets utility from all goods $j \in \left[i - \frac{x}{2}, i + \frac{x}{2}\right]$ where sums and subtractions are modulo 1.
- Agents are also indexed by the points on the unit circle and agent $a \in [0,1]$ always produces good $a$.

- Production costs one unit in effort and produces utility $u > 1$ to an agent who desires the good, and zero to others.
- Agents never consume their own production.
- Goods are perishable and they are produced on the spot, while money is non-perishable and can be carried from one period to another.
- As usual the agents discount future at rate $r$ and have a discount factor $\delta = \frac{1}{1+r}$.
- The agents meet randomly, and it is assumed that each agent meets someone in each period.
- The agents are anonymous, and in particular they do not observe each others' histories.

# Pure exchange economy

- Let us first study the economy without money.
- Any agent's life time utility evaluated at the end of a period is given by

$$U = \delta\left\{x^2(u-1) + (1-x^2)U\right\}$$

from which one easily solves

$$U = \delta\frac{x^2(u-1)}{1-\delta+\delta x^2}$$

- Assume that in the beginning of the economy proportion $m$ of the agents are endowed with one unit of money each.
- The agents can be in two states: Either an agent holds money or s/he is ready to produce a good.
- We denote the associated expected life time utilities by $V_m$ and $V_g$.
- Denote the probability that an agent accepts money in exchange for the good s/he produces by $\pi$.
- Denote the agent's belief that all the other agents accept money in exchange for a good by $\Pi$.
- The former is something the agent optimises given his/her belief about the behaviour in the economy.

- The value functions are determined by

$$V_g = \delta mx\, max_\pi \left[\pi \left(V_m - 1\right) + (1-\pi) V_g\right]$$

$$+\delta m(1-x)V_g + \delta(1-m)\left[x^2\left(u - 1 + V_g\right) + (1-x^2)V_g\right]$$

$$V_m = \delta\left\{mV_m + (1-m)\left[x\Pi\left(u + V_g\right) + (1 - x\Pi)V_m\right]\right\}$$

- In the asset valuation form the equations are

$$rV_g = mx\, max_\pi \pi\left(-1 + V_m - V_g\right) + (1-m)x^2\left(u-1\right) \qquad (1)$$

$$rV_m = (1-m)x\Pi\left(u + V_g - V_m\right) \qquad (2)$$

- The idea is to find an individually rational acceptance probability $\pi$ such that aggregating these probabilities into economywide probabilities $\Pi$ rationalises the individual's beliefs about the probability that money is accepted in trade.
- It is clear that there are only three truly different cases namely $\Pi = 0$, $\Pi = 1$ or $\Pi \in (0,1)$.
- Let us see when various decisions are optimal.

- Assume that $\Pi = 0$.
- Then an agent believes that once s/he has a unit of money then no-one accepts it.
- This clearly means that it is individually not optimal to accept money, or $\pi = 0$.
- When everyone optimises this way then no-one accepts money and, indeed, belief $\Pi = 0$ is vindicated as everyone holding money would just throw it away.
- Then the equilibrium would be identical to the equilibrium constructed when there is no money in the economy.
- Notice that if one wants to explain the use of money as an equilibrium phenomenon then it is very advisable that the model has an equilibrium where money is not used at all.

- Assume next that $\Pi = 1$.
- This is an equilibrium expectation only if it is individually optimal to accept money, or $-1 + V_m - V_g \geq 0$.
- Let us proceed with this and see what it implies.
- Instead of solving the value functions explicitly notice that subtracting (1) from (2) and solving we get

$$V_m - V_g = \frac{mx + (1-m)x^2 + x(1-x)(1-m)u}{r+x}$$

- So, the requirement that $V_m - V_g \geq 1$ is equivalent to

$$x(1-x)(1-m)(u-1) - r \geq 0$$

- This relation does not hold if $x$ is very small or if $m$ is very big.
- In the first case the double-coincidence of wants problem is so severe that even a single coincidence of wants problem is bad.
- This means that the costs of producing today cannot be covered by the future consumption as it takes so long to find a suitable good.
- In the latter case there are so many money holders that the agents very rarely meet good holders, and again the costs of producing today cannot be covered by the future consumption as it takes so long to find a suitable good.
- And, of course, a very large value of $r$ has a similar effect.

- Finally, let us assume that $\Pi \in (0,1)$.
- This equilibrium belief is vindicated only if each individual decision is $\pi = \Pi$.
- But if individuals mix between accepting money and not accepting it in equilibrium they have to be indifferent between the choices.
- This means that $-1 + V_m - V_g = 0$.
- Using (1) and (2) we now solve for

$$\Pi = \frac{r + (1-m)x^2(u-1)}{(1-m)x(u-1)} \tag{3}$$

- As $\Pi < 1$ we find that the equilibrium exists only if

$$x(1-x)(1-m)(u-1) - r > 0$$

- It is immediate that as $V_m = V_g + 1$ everyone who has a unit of money to start with will keep it (and not throw is away).

## Welfare

- Now that we know the conditions for the existence of a monetary equilibrium it is perhaps worthwhile to study whether the introduction of money increases welfare or not.
- There are two obvious measures of welfare.
- Either the expected utility of a randomly chosen agent $W(\pi)$ or the value created each period $N(\pi)$.
- The latter one equals the number of total trades times the value from one trade.

- The first one is given by

$$W(\pi = 0) = V_g = \frac{x^2(u-1)}{r}$$

$$W(\pi = 1) = mV_m + (1-m)V_g =$$

$$\frac{(1-m)x\,[m+(1-m)x]\,(u-1)}{r}$$

$$W(\pi \in (0,1)) = mV_m + (1-m)V_g =$$

$$\frac{(1-m)x^2(u-1)+mr}{r}$$

where we have utilised (3) and solved both $V_m$ and $V_g$ from (1) and (2).

- The total number of trades is given by

$$N(\pi = 0) = x^2(u-1)$$

$$N(\pi = 1) = \left[(1-m)(1-m)x^2 + (1-m)mx\right](u-1)$$
$$N(\pi \in (0,1)) = \left[(1-m)(1-m)x^2 + (1-m)mx\pi\right](u-1)$$

  where the derivation is quite straightforward.
- For instance, when money is accepted with probability one there are $1-m$ agents ready to produce.
- With probability $1-m$ they meet a similar agent and with probability $x^2$ both want what the other has and trade ensues.
- With probability $m$ these agents meet a money holder and with probability $x$ the money holder desires what the producer has leading to trade.
- The expressions for $W$ and $N$ are practically equivalent, and for that reason it is enough to focus on $W$.

- Note first that whenever the mixed strategy equilibrium and full monetary equilibrium exist for the same parameter values the latter provides higher welfare.
- Notice also that it could be that the barter economy provides higher welfare than the monetary economy.
- This happens when

$$\frac{x^2(u-1)}{r} > \frac{(1-m)x\left[m+(1-m)x\right](u-1)}{r}$$

or

$$x(2-m) > 1-m$$

- This is the case when trading is not so difficult, or $x$ is large, or when too many people hold money, or $m$ is too large.

- Because the agents can hold only one unit of anything it is possible that too much money is chasing around too few goods.
- One can see this in

$$\frac{\partial W(\pi = 1)}{\partial m} > 0$$

if and only if

$$x < \frac{1-2m}{2(1-m)}$$

- This does not hold if $m > \frac{1}{2}$ and for $m = 0$ it holds only if $x < \frac{1}{2}$ .

**Theorem**

*a) For all parameter values there exists an equilibrium where $m = 0$ and $\Pi = 0$. b) If $x(1-x)(1-m)(u-1) - r > 0$ and $m \in \left(0, \frac{x(1-x)(u-1)-r}{x(1-x)(u-1)}\right)$ both a pure monetary equilibrium and a mixed strategy equilibrium exist. c) If $x(1-x)(1-m)(u-1) - r = 0$ and $m \in \left(0, \frac{x(1-x)(u-1)-r}{x(1-x)(u-1)}\right)$ a pure monetary equilibrium exists.*

- The condition for $m$ is got by requiring that a producer does not resort to bargaining or $V_g \geq \frac{(1-m)x^2(u-1)}{r}$.

# Perfect bookkeeping

- One can contrast the monetary economy with one where there is perfect bookkeeping or all actions of the agents are public knowledge.
- There one can support an equilibrium where upon each meeting an agent produces if the other agent desires the good.
- If an agent deviates s/he is then doomed to autarky forever.
- Further, if any agent trades with such an agent s/he has the same fate.
- Now an agent can expect

$$rV_p = x^2(u-1) + x(1-x)u - x(1-x) = x(u-1)$$

- The equilibrium is viable if deviation does not pay.
- The crucial condition is that an agent who is supposed to produce without getting anything should find if profitable to do so.
- This means that

$$-1 + V_p \geq 0$$

or

$$r < x(u-1)$$

- This condition always holds if monetary equilibrium is possible to start with.
- It is also immediately clear that the welfare under the above equilibrium is higher than in any monetary equilibrium.

# Prices

- It is highly non-trivial to get prices in these models.
- One trick is to assume that production happens on the spot, and that the parties bargain over the amount to be produced.
- The agreed amount of production is then exchanged to a unit of indivisible money.
- Assume that the cost of producing is given by a convex function $c(q)$ which satisfies the Inada conditions.
- Assume that utility from consuming is given by a concave function $u(q)$ which satisfies the Inada conditions.
- Assume that these functions intersect at $q*$, and that at $\tilde{q}$ we have $c'(\tilde{q}) = u'(\tilde{q})$.

- When a money holder and a producer meet they use Nash-bargaining to determine the amount to be produced.
- Similarly, when two producers who want each others' good meet they use Nash-bargaining to determine the amount to be produced.
- Denote the agents' expectation about what is produced in the first type of meeting by $Q$ and in the second type of meeting by $R$.
- Now the value functions are given by

$$rV_g = (1-m)x^2\left(u(R) - c(R)\right) + mx\left(-c(Q) + V_m - V_g\right)$$

$$rV_m = (1-m)x\left(u(Q) + V_g - V_m\right)$$

- One can easily solve the value functions

$$
\begin{aligned}
V_g = {} & \frac{(1-m)x^2\left(r+(1-m)x\right)}{r(r+x)}\left(u(R)-c(R)\right) \\
& -\frac{mx\left(r+(1-m)x\right)}{r(r+x)}c(Q)+\frac{m(1-m)x^2}{r(r+x)}u(Q) \\
V_m = {} & \frac{(1-m)^2x^3}{r(r+x)}\left(u(R)-c(R)\right)-\frac{m(1-m)x^2}{r(r+x)}c(Q) \\
& \frac{(1-m)x\left[r(r+x)+m(1-m)x^2\right]}{r(r+x)\left(r+(1-m)x\right)}u(Q)
\end{aligned}
$$

- The bargaining problem when a money holder meets a commodity holder and the first one wants the latter one's good is

$$max_q\,(u(q)+V_g-V_m)(-c(q)+V_m-V_g)$$

- And when two commodity holders with double-coincidence of wants meet

$$max_\rho\,(u(\rho)-c(\rho))(u(\rho)-c(\rho))$$

- Notice that in both maximisation problems the value functions are calculated conditional on $Q$ and $R$.

- The first order conditions are

$$\frac{u'(q)}{u(q)+V_g-V_m}=\frac{c'(q)}{-c(q)+V_m-V_g} \tag{4}$$

$$\left(u'(\rho)-c'(\rho)\right)\left(u(\rho)-c(\rho)\right)=0 \tag{5}$$

- Notice that (5) defines $\rho=R=\tilde{q}$ or the efficienct solution.
- Equation 4 must define $q=Q$ from which one can derive conditions that have to be satisfied in such an equilibrium.
- If you figure out these conditions you should find out that $q=0$ is one possible equilibrium outcome.
- Then money has no value in the economy.
- When there is a solution $q>0$ then $q<\tilde{q}$.