Market structure

Seventh lecture

February 26, 2015

- Assume that we have similar setting, but static, as in the lecture on auctions, bargaining and posted prices.
- There are B buyers and S sellers.
- Now the sellers may be each in a different location as previously.
- Or they can be in the same location like in a market place.
- Assume that $B > S$, and that the sellers are in the same location.
- Then the sellers can ask a monopoly price $p = 1$.

- Assume that $B < S$, and that the sellers are in the same location.
- Then the situation is like Bertrand-competition, and the equilibrium price is $p = 0$.
- In this case each seller would rather be each in a location of his/her own, and the price would be $p = \frac{1-e^{-\theta}-\theta e^{-\theta}}{1-e^{-\theta}}$.
- Perhaps a more interesting possibility is demand uncertainty; there can be more or less buyers than sellers.
- First a simple example.

- There are two sellers 1 and 2 each with one unit of an identical good that they value at zero.
- There are three potential buyers; if a buyer gets a demand shock s/he wants to consume one unit.
- With probability p_i there are $i \in \{1, 2, 3\}$ buyers who want to consume.
- Each buyer is equally likely to get a demand shock.
- We study two market structures: A non-clustered market and clustered market.

- In a non-clustered market the sellers are in separate locations.
- Denote the prices by q_1 and q_2.
- The buyers' symmetric mixed strategy is the probability of contacting seller 1, $\pi(q_1, q_2)$.
- If a buyer gets a demand shock s/he updates the probabilities for demand

$$r_1 = \frac{\frac{1}{3}p_1}{\frac{1}{3}p_1 + \frac{2}{3}p_2 + (1 - p_1 - p_2)} = \frac{p_1}{3 - 2p_1 - p_2}$$

$$r_2 = \frac{2p_2}{3 - 2p_1 - p_2}$$

$$r_3 = \frac{3(1 - p_1 - p_2)}{3 - 2p_1 - p_2}$$

- A buyer's expected utility of visiting seller 1 is given by

$$r_1(1-q_1) + r_2\frac{\pi_1}{2}(1-q_1) + r_2(1-\pi_1)(1-q_1) +$$

$$r_3\frac{\pi_1^2}{3}(1-q_1) + r_3\frac{2\pi_1(1-\pi_1)}{2}(1-q_1) +$$

$$r_3(1-\pi_1)^2(1-q_1) \tag{1}$$

- A buyer's expected utility of visiting seller 2 is given by

$$r_1(1-q_2)+r_2\pi_1(1-q_2)+r_2\frac{1-\pi_1}{2}(1-q_2)+$$
$$r_3\pi_1^2(1-q_2)+r_3\frac{2\pi_1(1-\pi_1)}{2}(1-q_2)+$$
$$r_3\frac{(1-\pi_1)^2}{3}(1-q_2) \tag{2}$$

- Buyers' optimal behaviour is such that (1) = (2) from which one can by total differentiation determine

$$\frac{\partial \pi_1}{\partial q_1} = \frac{12r_1 + 9r_2 + 7r_3}{(1-q)(12r_2 + 16r_3)}$$

which is evaluated at the symmetric equilibrium $q_1 = q_2 = q$ resulting also in $\pi_1 = \frac{1}{2}$.

- Seller 1's objective is

$$max_{q_1} p_1 \pi_1 q_1 + p_2 \left(1 - (1-\pi_1)^2\right) q_1$$

$$+ (1 - p_1 - p_2) \left(1 - (1-\pi_1)^3\right) q_1$$

- Figuring out the first order condition and evaluating it at symmetric equilibrium $q_1 = q_2 = q$ one finds a candidate for the equilbrium price

$$q = \frac{(12r_2 + 16r_3)(4p_1 + 6p_2 + 7p_3)}{(12r_2 + 16r_3)(4p_1 + 6p_2 + 7p_3) + (12r_1 + 9r_2 + 7r_3)(8p_1 + 8p_2 + 6p_3)}$$

- Expected utility of a seller is

$$\frac{4p_1 + 6p_2 + 7p_3}{8} q$$

- In the clustered market the sellers are in the same location and buyers can visit both of them.

- It is clear that the sellers price using mixed strategies as long as $p_1 \in (0, 1)$.

- Denote the strategy by F and its support by $[l, L]$.

- First, it is clear that F is continuous, no mass points, and that $L = 1$.

- A seller who asks price l sells with probability one and consequently we must have $l = (1 - p_1)1$.
- A seller who asks price x sells with probability $p_1(1 - F(x)) + (1 - p_1)$ and we must have $p_1(1 - F(x))x + (1 - p_1)x = 1 - p_1$.
- From the previous we get $F(x) = \frac{x - 1 + p_1}{xp_1}$.

- It is simple but tedious to calculate that when $p_1 \in (0, 1)$ the sellers earn higher profits in the clustered than in the non-clustered market.
- In the example the demand may be 50%, 100% or 150% of the supply.
- It remain unclear to what extent the result generalises.
- So, let us study that next.

- Assume that the demand or measure of buyers θ comes from a continuous distribution H on $[0, m]$ where $m > 1$.
- Assume that there is a unit interval of sellers.
- Focus on symmetric equilibria and study non-clustered and clustered markets.
- Notice first that a buyer with a demand shock updates his/her belief about the level of demand as

$$g(a) = \frac{\frac{a}{m} h(a)}{\int_0^m \frac{x}{m} h(x) dx} = \frac{a h(a)}{E(\theta)}$$

- The timing is as follows: Sellers quote prices, nature chooses the demand, buyers observe prices and contact the sellers.

- Consider a non-clustered market where the Poisson-parameter θ, ratio of buyers to sellers, is stochastic.
- The probability that a seller meet k buyers in a symmetric equilibrium is $e^{-\theta}\frac{\theta^k}{k!}$.
- Denote the equilibrium price by q and assume that a seller deviates to price $\tilde{q}$.
- Using the market utility approach the queue length $\gamma = y\theta$ that a deviator expects is determined by

$$\int_0^m \frac{1-e^{-\gamma}}{\gamma} g(\theta)(1-\tilde{q}) = \int_0^m \frac{1-e^{-\theta}}{\theta} g(\theta)\theta(1-q)$$

- From this we can determine $\frac{\partial y}{\partial \tilde{q}} = -\frac{\int_0^m y(1-e^{-\gamma})h(\theta)}{(1-\tilde{q})\int_0^m (1-e^{-\gamma}-\gamma e^{-\gamma})h(\theta)}$ which tells how buyers behave.

- The seller's problem is

$$max_{\tilde{q}} \tilde{q} \int_0^m \left(1 - e^{-\gamma}\right) h(\theta) d\theta$$

- And the first order condition

$$\int_0^m \left(1 - e^{-\gamma}\right) h(\theta) + \tilde{q} \int_0^m e^{-\gamma} \theta \frac{\partial y}{\partial \tilde{q}} h(\theta) = 0$$

- Evaluated at $\tilde{q} = q$ where $y = 1$ one finds that

$$q = \frac{\int_0^m \left(1 - e^{-\theta} - \theta e^{-\theta}\right) h(\theta)}{\int_0^m \left(1 - e^{-\theta}\right) h(\theta)}$$

- In a clustered market things are quite simple.
- The sellers use a continuous mixed strategy F on $[1 - H(1), 1]$.
- $F(x) = H^{-1}\left(1 - \frac{1-H(1)}{x}\right)$.

- The expected profits in the different markets are got easily.
- In the clustered market each seller expects to get $1 - H(1)$.
- In the non-clustered market each seller expects to get $q \int_0^m \left(1 - e^{-\theta}\right) h(\theta) = \int_0^m \left(1 - e^{-\theta} - \theta e^{-\theta}\right) h(\theta)$.
- Clustered market profits are greater than non-clustered market profits if

$$\int_0^m \left(e^{-\theta} + \theta e^{-\theta}\right) h(\theta) > H(1)$$

- To gain more insight we allow the sellers to choose whether to go to a clustered or a non-clustered market.
- Assume that proportion σ of the seller go to the non-clustered market and buyers go there with probability z.
- Then the expected queue length there is $\gamma = \frac{z\theta}{\sigma}$.
- Now the formulae for equilibrium price and all related magnitudes remain except that queue length θ is replace by γ.
- One can determine the expected utility of a buyer

$$\frac{1}{E(\theta)} \int_0^m \theta e^{-\gamma} h(\theta) \qquad (3)$$

- It is more complicated to determine a buyer's expected utility in a clustered market.
- We use the following trick.
- Assume that all sellers ask the same price p.
- Determine the price p such that the sellers' expected utility is the same as under the mixed strategy F where now $F(1) = 1 - \sigma$.
- We know the sellers' expected utility under F to be $1 - H\left(\frac{1-\sigma}{1-z}\right)$.
- The price turns out

$$p = \frac{1 - H\left(\frac{1-\sigma}{1-z}\right)}{1 - \int_0^{(1-\sigma)/(1-z)} \frac{1-\sigma}{1-z} H(\theta) d\theta}$$

- Exactly the same number of trades are consummated in the clustered market under F and under virtual price p.
- Consequently, the buyers' expected utility under p must be the same as under F.
- This is

$$\frac{1}{E(\theta)}\frac{1-\sigma}{1-z}\left[H\left(\frac{1-\sigma}{1-z}\right)-\int_0^{(1-\sigma)/(1-z)}\frac{1-\sigma}{1-z}H(\theta)d\theta\right] \quad (4)$$

- Now that we have an expression for the buyers' expected utility in both markets we can equate them as the equality determines which proportion of buyers goes to which market.
- (3) = (4) is equivalent to

$$\int_0^{(1-\sigma)/(1-z)}\theta h(\theta)d\theta = \int_0^{m}\theta e^{-\gamma}h(\theta)d\theta \quad (5)$$

- The sellers do better in the clustered market if

$$1 - H\left(\frac{1-\sigma}{1-z}\right) > \int_0^m \left(1 - e^{-\gamma} - \gamma e^{-\gamma}\right) h(\theta) d\theta \qquad (6)$$

- Substituting from (5) an equivalent form of (6) is

$$\int_0^m e^{-\gamma} h(\theta) d\theta + \int_0^{(1-\sigma)/(1-z)} \frac{z}{\sigma} \theta h(\theta) d\theta > \int_0^{(1-\sigma)/(1-z)} h(\theta) d\theta$$

which is equivalent to

$$\int_0^{(1-\sigma)/(1-z)} \left(\frac{z}{\sigma}\theta - 1 + e^{-\frac{z}{\sigma}\theta}\right) h(\theta) d\theta + \int_{(1-\sigma)/(1-z)}^m e^{-\gamma} h(\theta) d\theta > 0$$

- But $x - 1 + e^{-x} > 0$ for all $x > 0$.

- This means that whenever the buyers behave 'subgame optimally' the sellers who are in the clustered market do better.
- This also means that the clustered and non-clustered market cannot co-exist.
- Thus, there are two equilibria: In one everybody is in a clustered market, in the other everybody is in the non-clustered market.
- There are many ways to argue that one of the above equilibria is more plausible than the other.
- One is to use replicator dynamics like in Lu and McAfee (1996).

- The non-clustered equilibrium does not survive standard equilibrium refinements.
- Assume that sellers make a mistake in locating with probability $\varepsilon > 0$.
- Thus, even when they want play the non-clustered equilibrium proportion ε of them end up in the clustered market.
- Suppose that proportion v of the buyers goes to the clustered market.
- Proportion $(1-\varepsilon)v + \varepsilon(1-v)$ actually ends up there.
- This is larger than v if $v < \frac{1}{2}$ which certainly holds for small ε.
- Then the sellers in the clustered market would do better than in the non-clustered market.
- Thus, the sellers would intentionally 'make mistakes'.