Shimer's AER-article (2005)

Eighth lecture

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- We study the main points of Shimer's 2005 AER-article.
- The objective of the article is to determine whether a Pissarides-Mortensen type labour-search model can account for observed behaviour of the key variables.
- The result is that the model cannot explain the cyclical behaviour of unemployment and vacancies.
- Neither does the behaviour of wages correspond to empirical findings.
- The fluctuations in the model are smaller than in the data, and it does not have strong propagation mechanisms.
- The reason is that Nash-bargaining dampens the shocks to labour productivity.
- As a result the firms do not create vacancies, and there is not much change in the endogenous variables.

Some observations about labour markets

- The unemployment rate u is countercyclical and volatile.
- The vacancy measure v is procyclical, and more so than unemployment rate countercyclical.
- The job market tightness $\theta = \frac{v}{u}$ is very procyclical.
- The Beveridge curve is downward sloping.
- Job destruction rate varies from industry to industry and in services but is mostly countercyclical.

- As the focus is on pro- and counter cyclicality of the variables one has to make the model stochastic first.
- This is done by postulating exogenous variables that are the driving force behind fluctuations.

- We first list the main variables of the model.

1. p labour productivity follows first order Markov process in continuous time.
2. s separation rate follows first order Markov process in continuous time.
3. λ the Poisson-rate at which a shock hits the economy.

- When a shock hits a new pair (p', s') is determined from a state dependent distribution.

- Other variables and notation.
- $\mathbb{E}_{p,s}X_{p',s'} \equiv \mathbb{E}(X((p',s')|(p,s))$ for any variable X.
- $[0,1]$ unit interval of workers who are risk-neutral and infinitely lived.
- $[0,A]$a sufficiently large interval of firms that are risk-neutral and infinitely lived.
- r common discount factor.
- z flow benefit of an unemployed worker.
- $p(t)$ stochastic labour productivity at time t.
- c flow cost of keeping a vacancy open.

- $s(t)$ stochastic separation rate at time t.
- $u(t)$ unemployment rate at time t.
- $v(t)$ measure of vacancies at time t.
- $\theta(t) = \frac{v(t)}{u(t)}$ job market tightness at time t.
- $m(u(t), v(t))$ constant returns to scale flow of matches function at time t; increasing in both arguments.
- $f(\theta(t)) = m(1, \theta(t))$ job finding rate at time t.
- $q(\theta(t)) = m\left(\frac{1}{\theta(t)}, 1\right)$ vacancy filling rate at time t.
- $\beta \in (0,1)$ worker's bargaining power in Nash-bargaining.

- **Assumptions**

1. The economy proceeds in continuous time.
2. The current values of p and s are always common knowledge.
3. Firms have constant returns to scale production technology using only labour.
4. Productivity $p(t) > z$ in all states.
5. Wage is determined by Nash-bargaining when an unemployed and a vacancy first meet.
6. No stand is taken what happens when the state changes except that possible negotiation of wage results in an efficient outcome.

Equilibrium

- Focus on equilibrium where the labour market tightness depends only on the current values of p and s.
- The objective is to determine how u, v and wage evolve.
- The unemployment rate evolves according to

$$\dot{u}(t) = s(t)(1 - u(t)) - f\left(\theta_{p(t),s(t)}\right)u(t) \tag{1}$$

- Note first that the arguments are given as subscripts presumably to lighten the notation.
- $p(t)$ and $s(t)$ signify the aggregate state at time t which makes the mapping to data sensible.
- The interpretation of (1) is straightforward.
- The LHS is the time derivative of unemployment.
- The RHS tells that it consists of the flow of employed who lose their job (increase in unemployment) and unemployed who find a job (decrease).

- Unlike usually here it is enough to determine just one value function, the joint expected life time surplus of a worker and a firm that are matched.
- We assume that the behaviour of the agents depends only on the state not on the time.
- To derive the relation we need the corresponding value functions of an unemployed worker, employed worker and a filled vacancy/job.

$$rU_{p,s} = z + f(\theta_{p,s})(E_{p,s} - U_{p,s}) + \lambda\left(\mathbb{E}_{p,s}U_{p',s'} - U_{p,s}\right) \quad (2)$$

$$rE_{p,s} = w_{p,s} - s(E_{p,s} - U_{p,s}) + \lambda\left(\mathbb{E}_{p,s}E_{p',s'} - E_{p,s}\right) \quad (3)$$

$$rJ_{p,s} = p - w_{p,s} - sJ_{p,s} + \lambda\left(\mathbb{E}_{p,s}J_{p',s'} - J_{p,s}\right) \quad (4)$$

- The interpretation of these equations should be clear by now
- For instance, in (4) the return to having a worker is p while the firm has to pay $w_{p,s}$. With 'probability' s the partnership is dissolved and the capital gain of the firm is $0 - J_{p,s}$. With probability λ the state changes and the capital gain is $\mathbb{E}_{p,s} J_{p',s'} - J_{p,s}$.
- To determine the joint surplus of the worker and the firm we define $V_{p,s} \equiv J_{p,s} + E_{p,s} - U_{p,s}$, and summing (3) and (4) and subtracting (2) we get

$$rV_{p,s} = p - z - f\left(\theta_{p,s}\right)\left(E_{p,s} - U_{p,s}\right) - sV_{p,s} + \lambda\left(\mathbb{E}_{p,s} V_{p',s'} - V_{p,s}\right) \tag{5}$$

- The Nash-bargaining leads to wage that maximises

$$(E_{p,s} - U_{p,s})^{\beta} J_{p,s}^{1-\beta} \tag{6}$$

- The FOC to this problem implies

$$\frac{E_{p,s} - U_{p,s}}{\beta} = V_{p,s} = \frac{J_{p,s}}{1-\beta} \tag{7}$$

- The middle equality in (7) is got by solving $J_{p,s} = \frac{1-\beta}{\beta}(E_{p,s} - U_{p,s})$.
- Then $V_{p,s} = J_{p,s} + E_{p,s} - U_{p,s} = \frac{1-\beta}{\beta}(E_{p,s} - U_{p,s}) + E_{p,s} - U_{p,s} = \frac{E_{p,s}-U_{p,s}}{\beta}$.
- Inserting this into (5) yields

$$rV_{p,s} = p - (z + f(\theta_{p,s}) \beta V_{p,s}) - sV_{p,s} + \lambda \left(\mathbb{E}_{p,s} V_{p',s'} - V_{p,s}\right) \tag{8}$$

- Free entry of the firms provides yet another relation

$$q(\theta_{p,s})(1-\beta)V_{p,s} - c = 0 \tag{9}$$

where the first term is what a vacancy expects to gain and c is the cost of keeping the vacancy open.

- We use (9) to eliminate $V_{p,s}$-terms in (8)

$$\frac{r+s+\lambda}{q(\theta_{p,s})} + \beta\theta_{p,s} = (1-\beta)\frac{p-z}{c} + \lambda\mathbb{E}_{p,s}\frac{1}{q(\theta_{p',s'})} \tag{10}$$

where we have utilised the facts that (9) holds for all times and states, and that $f(\theta_{p,s})/q(\theta_{p,s}) = \theta_{p,s}$.

- Expression (10) implicitly defines the job market tightness or v-u-ratio as a function of the current state (p, s).

Some comparative statics

- Shimer first analyses (10) by making some simplifying assumptions so as to get results non-numerically.
- Assume that there are no aggregate shocks or $\lambda = 0$.
- Then (10) becomes

$$\frac{r+s}{q(\theta_{p,s})} + \beta\theta_{p,s} = (1-\beta)\frac{p-z}{c} \tag{11}$$

- Remember that elasticity of a function $g(x)$ is defined by $\frac{g'(x)}{g(x)/x}$.
- Notice also that $q(\theta_{p,s}) = \frac{1}{\theta_{p,s}} f(\theta_{p,s})$.

- Totally differentiate (11) with respect to $\theta_{p,s}$ and $p-z$ to get

$$d\theta_{p,s}\left\{\frac{r+s}{f}-\frac{(r+s)\theta_{p,s}}{f^2}f'+\beta\right\}$$

$$-d(p-z)\frac{1-\beta}{c}=0 \tag{12}$$

from which we find

$$\frac{d\theta_{p,s}}{d(p-z)}=\frac{\frac{1-\beta}{c}}{\frac{r+s}{f}-\frac{r+s}{f}\eta+\beta} \tag{13}$$

where $\eta=\frac{f'}{f/\theta_{p,s}}$.

- The elasticity of $\theta_{p,s}$ with respect to the net productivity is given by $\frac{d\theta_{p,s}}{d(p-z)}\frac{p-z}{\theta_{p,s}}$. Solving $\frac{p-z}{c}$ from (11) and inserting it into the formula for elasticity we get

$$\frac{d\theta_{p,s}}{d(p-z)}\frac{p-z}{\theta_{p,s}} = \frac{r+s+\beta f}{(r+s)(1-\eta)+\beta f} \tag{14}$$

- Shimer's objective is to study the sensitivity of this magnitude to various assumptions.
- He finds that with 'reasonable' parameter values it is close to unity, and on top of that the magnitudes required to make it, say, greater than 2 are not plausible.
- Altogether, the job market tightness is unresponsive to changes in (labour) productivity.

- Analogous exercise of figuring the elasticity of the v-u-ratio with respect to separation rate produces

$$\frac{d\theta_{p,s}}{ds}\frac{s}{\theta_{p,s}} = \frac{-s}{(r+s)(1-\eta)+\beta f} \tag{15}$$

- This turns out to show similar insensitivity of the v-u-ratio to changes in parameters as (14).
- $(11)-(14)$ come from the optimal behaviour of the agents.
- A so called independent relationship of vacancies and unemployment is deducible from (1) with $\dot{u}=0$ which is the steady-state condition.

- If one specifies that the matching function is of Cobb-Douglas type $m(u,v) = \mu u^{\alpha} v^{1-\alpha}$ then (1) implies $s(1-u) - \mu 1^{\alpha} \theta^{1-\alpha} u = 0$ and as $\theta = \frac{v}{u}$ one gets (adding indeces)

$$v_{p,s} = \left(\frac{s\,(1 - u_{p,s})}{\mu\, u_{p,s}^{\alpha}} \right)^{\frac{1}{1-\alpha}} \tag{16}$$

- For a fixed s this is a decreasing relationship between vacancies and unemployment and fits the empirical Beveridge-curve.
- Vacancies and unemployment should move in opposite directions in response to shocks that increase labour productivity.
- Increase in the separation rate does not affect the v-u-ratio much; it increases both vacancies and unemployment.
- These preliminary calculations indicate what one can expect of the simulation exercises.

- Shimer performs several other comparative statics analyses, for instance, postulating meeting rates assuming contacting by only firms and by only workers.
- The end result is that unless the bargaining power of the workers is extreme the v-u-ratio is not much affected.
- Calibration exercise uses Cobb-Douglas matching function and so called HP-filtering to uncover the trend from the simulated data.
- In the exercise either labour productivity is stochastic or the separation rate is stochastic.
- The results can be read from tables 1, 3 and 4. In table 3 the standard deviation of v/u as well as in f is very low, meaning that there is little volatility.
- The data in table 1 shows volatility that is at least 10 times higher for v/u, and 12 times higher for f.
- Also the correlation between labour productivity p, v/u and f is about unity in the simulated model (table 3) while in the data there are big differences.

- In table 4 the separation rate is stochastic, and one can see that the correlation between u and v is practically perfect meaning that there is no variation in v/u.
- This cannot be found in the data.
- Wages have not played any role so far.
- Let us assume that they are determined by Nash-bargaining whenever a shock takes place.
- Doing the same kind of substitutions as in the derivation of $V_{p,s}$ (see appendix B of the article) one finds that the wage equation solves

$$w_{s,p} = (1-\beta)z + \beta\left(p + c\theta_{p,s}\right) \tag{17}$$

- One can see that an increase in s causes a small decline in v/s (table 4) which then decreases wages.
- Thus, even though increase in s is bad for the firms the lower wages off-set some of this effect.
- Similarly, an increase in productivity goes to a large part to the wages, and lowers the firms incentives to create vacancies.
- The content of Shimer's critique is that the standard search model does not feature the strong pro- cyclicality of v/u and the job finding rate with simultaneously weak procyclicality of labour productivity.
- The elasticities wrt p generated by the model are far too small.

- Hall (AER 2005) has a very similar model in which he imposes wage rigidity.
- It then fits the data better.
- Hall and Milgrom (AER 2008) utilise alternating offer bargaining the way Binmore-Rubinstein-Wolinsky develop it.
- In this setting there is a difference between continuing bargaining but not agreeing and ending bargaining.
- The difference is in the outside options: If a party rejects and offer and considers making a counter offer the parties get a disagreement pay-off. If a party abandons bargaining the parties get their expected life time utilities of searching.
- This makes wage less sensitive to productivity shocks, and the model matches the data better.

- Ljungqvist and Sargent have a recent article where they develop a concept of fundamental surplus which is supposed to tell to what extent there are pro- and counter cyclical movements in any search model http://www.hecer.fi/images/documents/papers/ljungqvist_300115.p