

Introduction to Quantum fields in Curved Spaces

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Solvalla

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- 1 Introduction
- 2 Cosmological particle creation
- 3 Black hole evaporation
- 4 Electroweak vacuum stability
- 5 Summary

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Fundamental concepts emerge

1900 – 1927

- 1 Extending classical mechanics becomes unavoidable
 - Black-body radiation
 - Photoelectric effect
 - The Bohr model for hydrogen
 - ...

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- 1 Extending classical mechanics becomes unavoidable
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 - ...
- 2 Special Relativity needs to be incorporated
 - Relativistic quantum mechanics has severe issues

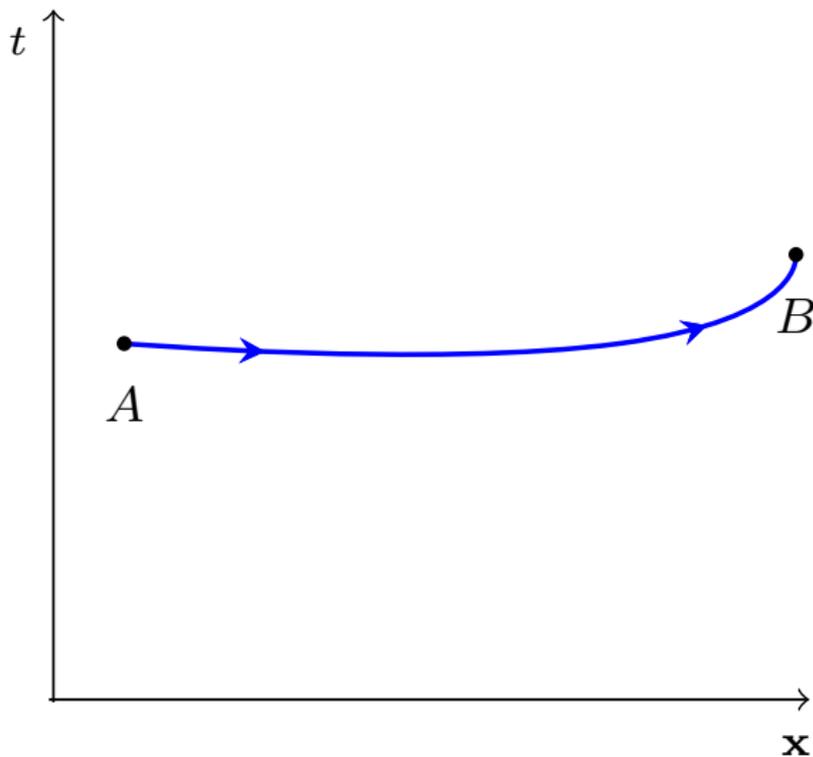
Fundamental concepts emerge

1900 – 1927

- 1 Extending classical mechanics becomes unavoidable
 - Black-body radiation
 - Photoelectric effect
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 - ...
- 2 Special Relativity needs to be incorporated
 - Relativistic quantum mechanics has severe issues
- 3 1927 \Rightarrow Quantum field theory
 - Particles can be created and annihilated by the vacuum

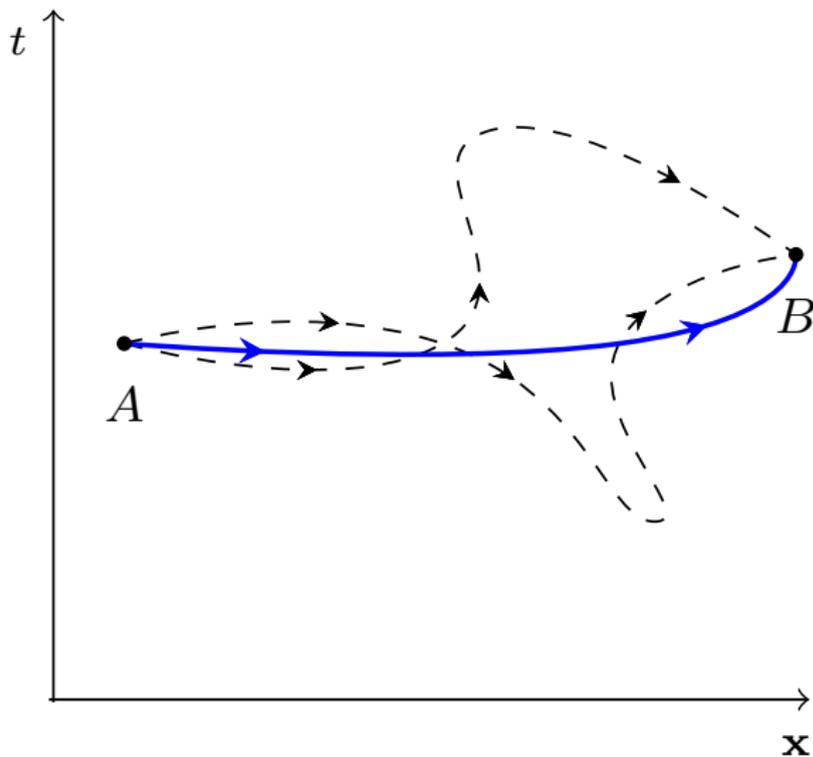
Classical Mechanics

- Deterministic



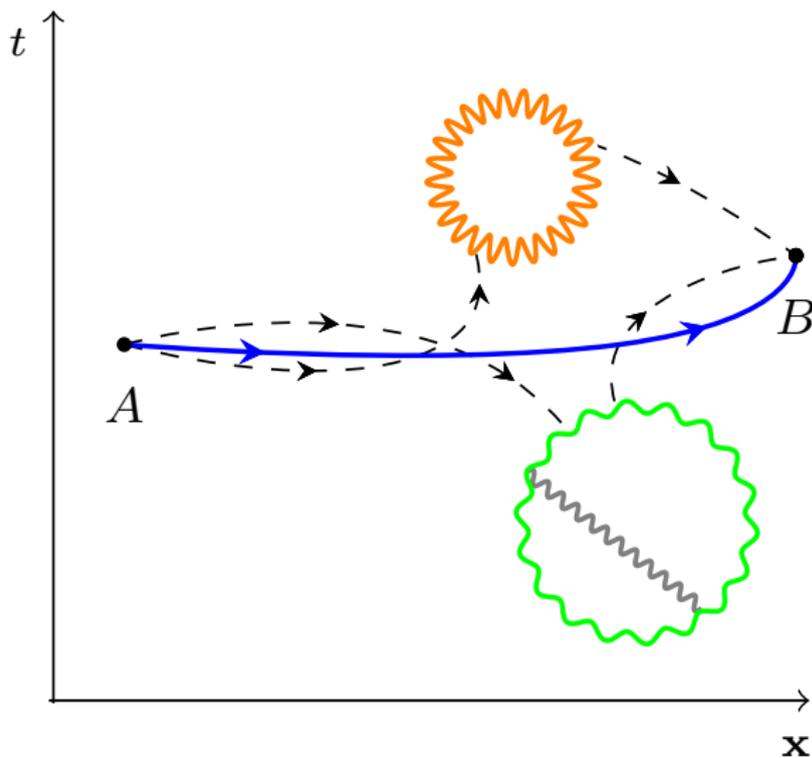
Quantum Mechanics

- Probabilistic



Quantum Field Theory

- Probabilistic with particle creation/annihilation



Renormalization

- The Standard model of particle physics is a very successful QFT
 - **Electrodynamics**
 - **Strong** nuclear force
 - **Weak** nuclear force

Standard Model of Elementary Particles

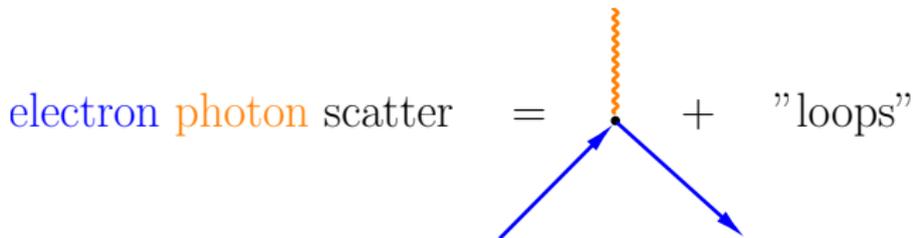
		three generations of matter (fermions)						
		I	II	III				
mass		+2.4 MeV/c ²	+1.275 GeV/c ²	+172.44 GeV/c ²	0		+125.09 GeV/c ²	
charge		2/3	2/3	2/3	0		0	
spin		1/2	1/2	1/2	1		0	
		u up	c charm	t top	g gluon		H Higgs	
	QUARKS							SCALAR BOSONS
		+4.8 MeV/c ²	+95 MeV/c ²	+4.18 GeV/c ²	0			
		-1/3	-1/3	-1/3	0			
		1/2	1/2	1/2	1			
		d down	s strange	b bottom	γ photon			
		+0.511 MeV/c ²	+105.67 MeV/c ²	+1.7768 GeV/c ²	0		+91.19 GeV/c ²	
		-1	-1	-1	0			
		1/2	1/2	1/2	1			
		e electron	μ muon	τ tau	Z Z boson			
	LEPTONS							GAUGE BOSONS
		<2.2 eV/c ²	<1.7 MeV/c ²	<15.5 MeV/c ²	0		+80.39 GeV/c ²	
		0	0	0	0			
		1/2	1/2	1/2	1/2			
		ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson			

Renormalization

- The Standard model of particle physics is a very successful QFT
 - **Electrodynamics**
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- **Renormalizability** is a crucial requirement
 - QFTs are plagued by infinities that need to be consistently dealt with

Standard Model of Elementary Particles

three generations of matter (fermions)					
	I	II	III		
mass	$\approx 2.4 \text{ MeV}/c^2$	$\approx 1.275 \text{ GeV}/c^2$	$\approx 172.44 \text{ GeV}/c^2$	0	$\approx 125.09 \text{ GeV}/c^2$
charge	$2/3$	$2/3$	$2/3$	0	0
spin	$1/2$	$1/2$	$1/2$	1	0
	u up	c charm	t top	g gluon	H Higgs
QUARKS					SCALAR BOSONS
	$\approx 4.8 \text{ MeV}/c^2$	$\approx 95 \text{ MeV}/c^2$	$\approx 4.18 \text{ GeV}/c^2$	0	
	$-1/3$	$-1/3$	$-1/3$	0	
	$1/2$	$1/2$	$1/2$	1	
	d down	s strange	b bottom	γ photon	
					GAUGE BOSONS
	$\approx 0.511 \text{ MeV}/c^2$	$\approx 105.67 \text{ MeV}/c^2$	$\approx 1.7768 \text{ GeV}/c^2$	$\approx 91.19 \text{ GeV}/c^2$	
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LEPTONS					
	$\approx 2.2 \text{ eV}/c^2$	$\approx 1.7 \text{ MeV}/c^2$	$\approx 15.5 \text{ MeV}/c^2$	$\approx 80.39 \text{ GeV}/c^2$	
	0	0	0	± 1	
	$1/2$	$1/2$	$1/2$	1	
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	



Issues with Quantum Gravity

- A major problem in General Relativity as a QFT:
Non-Renormalizability
- ⇒ The "usual" approach of QFT fails!
- Non-perturbative theories are not well-behaved as loop expansions
 - Questionable predictability

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- ⇒ The "usual" approach of QFT fails!
- Non-perturbative theories are not well-behaved as loop expansions
 - Questionable predictability
- Currently many ideas are explored
 - String Theory
 - Loop Quantum Gravity
 - Supergravity
 - Non-commutative spacetime
 - ...

Quantum field theory in curved space

- A middle ground: Quantize everything except gravity
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- **Example:** a (quantized) scalar field with background **gravity**

$$S = - \int d^4x \sqrt{|g|} \left[\frac{1}{2} \nabla^\mu \hat{\phi} \nabla_\mu \hat{\phi} + \frac{1}{2} m^2 \hat{\phi}^2 + \frac{\xi}{2} R \hat{\phi}^2 + \frac{\lambda}{4} \hat{\phi}^4 \right]$$

(in flat space: $g \rightarrow 1$, $\nabla_\mu \rightarrow \partial_\mu$ and $R \rightarrow 0$)

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(in flat space: $g \rightarrow 1$, $\nabla_\mu \rightarrow \partial_\mu$ and $R \rightarrow 0$)

- 1 Allows the study of how curvature effects quantum observables
- 2 Allows the study of *backreaction*
 - Semi-Classical Einstein equation ($M_{\text{P}}^2 \equiv 1$):

$$\underbrace{G_{\mu\nu}}_{\text{classical}} = \underbrace{\langle \hat{T}_{\mu\nu} \rangle}_{\text{quantum}}$$

General remarks

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 - Nowadays not so much...
 - Interactions were rarely discussed

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Important applications

- Black hole evaporation (70's)
- Early Universe physics
 - Inflation (quite recent)
 - Vacuum stability of the SM (very recent)

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Textbooks:

- 1 **Birrell & Davies:**
Quantum Fields in Curved Space
- 2 **Parker & Toms:**
Quantum Field Theory in Curved Spacetime
- 3 **Mukhanov & Winitzki:**
Introduction to Quantum Effects in Gravity

Cosmological particle creation

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General remarks

- Assume a homogeneous and isotropic spacetime, but which may expand or contract (FLRW)
- The *metric*: $g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a(t)d\mathbf{x}^2$

The scale factor $a(t)$

- Characterizes cosmic expansion
- $\frac{l(t_2)}{l(t_1)} = \frac{a(t_2)}{a(t_1)}$

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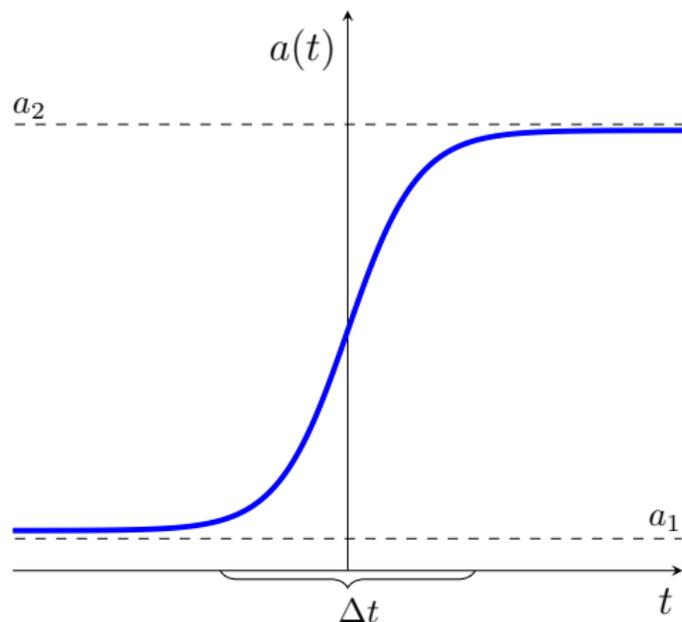
The scale factor $a(t)$

- Characterizes cosmic expansion
 - $\frac{l(t_2)}{l(t_1)} = \frac{a(t_2)}{a(t_1)}$
- The concepts of particle and vacuum are not global
i.e. they are observer dependent
 - First proper treatment in [Parker \(69\)](#)

Cosmological particle creation (Example)

Example:

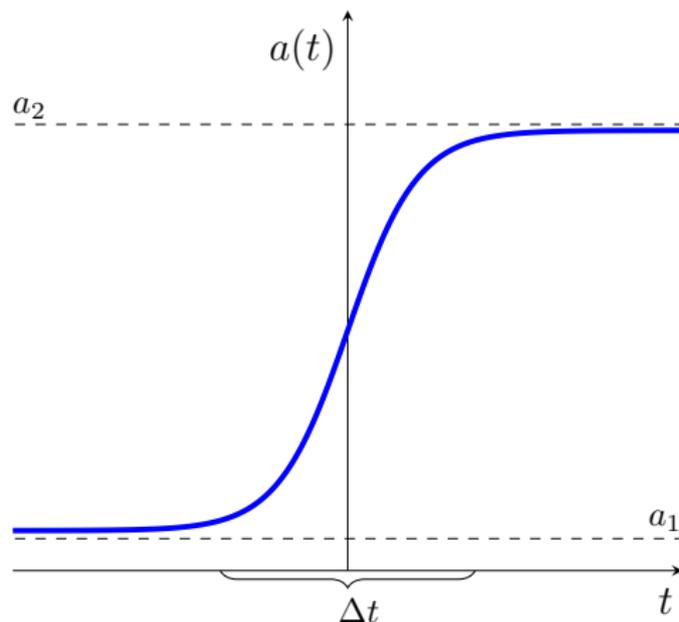
$$a(t) = \frac{a_1 + a_2}{2} + \frac{a_2 - a_1}{2} \tanh\left(\frac{t}{\Delta t}\right); \quad \begin{cases} a(-\infty) = a_1 \\ a(+\infty) = a_2 \end{cases}$$



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The "in" quantum mode

$$f_{\mathbf{k}}^{\text{in}}(t) = \frac{\exp\{-i\omega_+ t - i\Delta t \omega_- \ln[2 \cosh(t/\Delta t)]\}}{\sqrt{2\omega_{\text{in}}}} \\ \times {}_2F_1\left[1 + i\Delta t \omega_-, i\Delta t \omega_-; 1 - i\Delta t \omega_{\text{in}}\right. \\ \left.; \frac{1 + \tanh(t/\Delta t)}{2}\right] \xrightarrow{t \rightarrow -\infty} e^{-i\omega_{\text{in}} t}$$

$$\omega_{\text{in/out}}^2 = \mathbf{k}^2 + a_1^2/2 m^2; \quad \omega_{\pm} = \frac{\omega_{\text{out}} \pm \omega_{\text{in}}}{2}$$

Cosmological particle creation (continued)

- Energy density changes from vacuum to thermal:

$$\frac{E(t \rightarrow \infty)}{V} - \frac{E(t \rightarrow -\infty)}{V} = \int \frac{d^3k}{(2\pi)^2} \omega e^{-\omega/T}; \quad T \sim \Delta t$$

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- "Quantum friction"

Lenz's Law:

The outcome is always such as to oppose the change producing it.

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- Early work on cosmological particle creation laid the foundation for Hawking's calculation

Black Hole evaporation

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Particle Creation by Black Holes

S. W. Hawking

Department of Applied Mathematics and Theoretical Physics, University of Cambridge,
Cambridge, England

Received April 12, 1975

Abstract. In the classical theory black holes can only absorb and not emit particles. However it is shown that quantum mechanical effects cause black holes to create and emit particles as if they were hot bodies with temperature $\frac{\hbar\kappa}{2\pi k} \approx 10^{-6} \left(\frac{M_{\odot}}{M}\right) \text{°K}$ where κ is the surface gravity of the black hole. This thermal emission leads to a slow decrease in the mass of the black hole and to its eventual disappearance: any primordial black hole of mass less than about 10^{15} g would have evaporated by now. Although these quantum effects violate the classical law that the area of the event horizon of a black hole cannot decrease, there remains a Generalized Second Law: $S + \frac{1}{4}A$ never decreases where S is the entropy of matter outside black holes and A is the sum of the surface areas of the event horizons. This shows that gravitational collapse converts the baryons and leptons in the collapsing body into entropy. It is tempting to speculate that this might be the reason why the Universe contains so much entropy per baryon.

8. Gauge theory correlators from noncritical string theory

(7650) S.S. Gubser, Igor R. Klebanov, Alexander M. Polyakov (Princeton U.). Feb 1998. 14 pp.

Published in **Phys.Lett. B428 (1998) 105-114**

PUPT-1767

DOI: [10.1016/S0370-2693\(98\)00377-3](https://doi.org/10.1016/S0370-2693(98)00377-3)

e-Print: [hep-th/9802109](https://arxiv.org/abs/hep-th/9802109) | [PDF](#)

[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)

[ADS Abstract Service](#); [AMS MathSciNet](#)

[Detailed record](#) - Cited by 7650 records **1000+**

9. Partial Symmetries of Weak Interactions

(7169) S.L. Glashow (Copenhagen U.). 1961. 10 pp.

Published in **Nucl.Phys. 22 (1961) 579-588**

DOI: [10.1016/0029-5582\(61\)90469-2](https://doi.org/10.1016/0029-5582(61)90469-2)

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10. Particle Creation by Black Holes

(7059) S.W. Hawking (Cambridge U., DAMTP & Caltech). Aug 1975. 22 pp.

Published in **Commun.Math.Phys. 43 (1975) 199-220**, Erratum: **Commun.Math.Phys. 46 (1976) 206**

DOI: [10.1007/BF01608497](https://doi.org/10.1007/BF01608497), [10.1007/BF02345020](https://doi.org/10.1007/BF02345020)

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[AMS MathSciNet](#); [Project Euclid](#)

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11. The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problem

(6789) Alan H. Guth (SLAC). Jul 1980. 32 pp.

Published in **Phys.Rev. D23 (1981) 347-356**

SLAC-PUB-2576

DOI: [10.1103/PhysRevD.23.347](https://doi.org/10.1103/PhysRevD.23.347)

[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)

[KEK scanned document](#); [ADS Abstract Service](#); [OSTI.gov Server](#); [SLAC Document Server](#); [Link to Full](#)

[Detailed record](#) - Cited by 6789 records **1000+**

12. Asymptotic Freedom in Parton Language

(6342) Guido Altarelli (Ecole Normale Supérieure), G. Parisi (IHES, Bures-sur-Yvette). Mar 1977. 21 pp.

Published in **Nucl.Phys. B126 (1977) 298-318**

LPTENS-77-6

DOI: [10.1016/0550-3213\(77\)90384-4](https://doi.org/10.1016/0550-3213(77)90384-4)

[References](#) | [BibTeX](#) | [LaTeX\(US\)](#) | [LaTeX\(EU\)](#) | [Harvmac](#) | [EndNote](#)

[Detailed record](#) - Cited by 6342 records **1000+**

Black holes, classically

- Bardeen et al. (73) & Bekenstein (73):

Black holes have entropy?

- Dynamics similar to first law of thermodynamics
- Area cannot decrease i.e second law of thermodynamics

$$A_1 + A_2 \leq A_{12}$$

- Degradation of energy (cannot be transformed into work)
- Information loss

$$S = - \sum_n p_n \log p_n$$

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- Bekenstein (73): Black hole entropy

$$S \propto A$$

A crucial question:

How can a black hole have temperature!?

$$\delta Q = T dS$$

Hawking radiation

- Hawking (74,75): Black holes radiate thermally
 - Computationally similar to cosmological particle creation

$$T = \frac{c^3 \hbar}{8\pi GM k_B}$$

- Assuming thermodynamics this fixes the entropy

$$dU = TdS - pdV \quad \Rightarrow \quad S = \frac{c^3 k_B A}{4\hbar G}$$

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- One of the few glimpses we have into quantum nature of gravity
- A complete microscopic description does not yet exist

Common misconceptions

- ✗ "Particles that have once fell in can tunnel out."
- ✗ "Antiparticles fall into the hole making it shrink."

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Thus it will give positive energy flux out across the event horizon or, equivalently, a negative energy flux in across the event horizon.

Important features

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- The produced spectrum does not depend on the infallen matter

The information loss paradox

- Introduced in [Hawking \(76\)](#)

Another misconception

x "Since the system contains a thermally distributed particle number it *is* thermal."

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- No(!) information paradox in cosmological particle creation
- The key ingredient is the emergence of an *event horizon*

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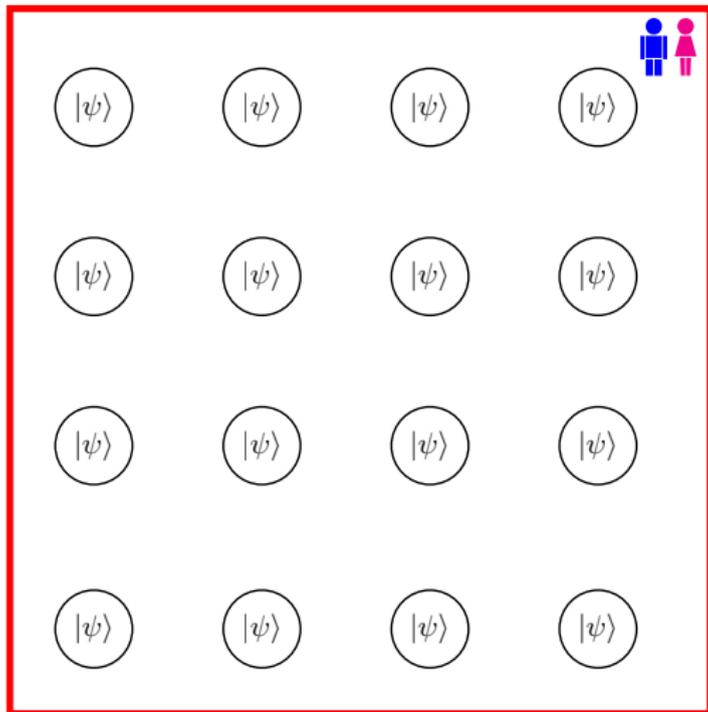
✗ "Since the system contains a thermally distributed particle number it *is* thermal."

- No(!) information paradox in cosmological particle creation
- The key ingredient is the emergence of an *event horizon*
- Implies non-Unitary evolution
 - Probability is not conserved
 - A pure states evolves into a mixed one
 - Should not occur in QFT

$$|\psi\rangle \longrightarrow \hat{\rho}; \quad \hat{\rho}^2 \neq \hat{\rho}$$

Initially, complete information

The observable Universe

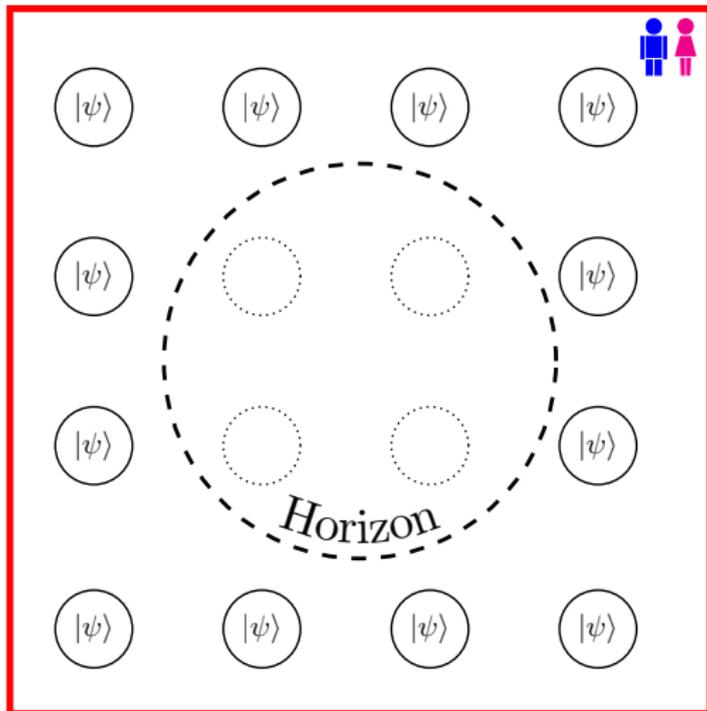


State of the system:

$$|\Psi\rangle$$

Information loss from the cosmological Horizon

The observable Universe



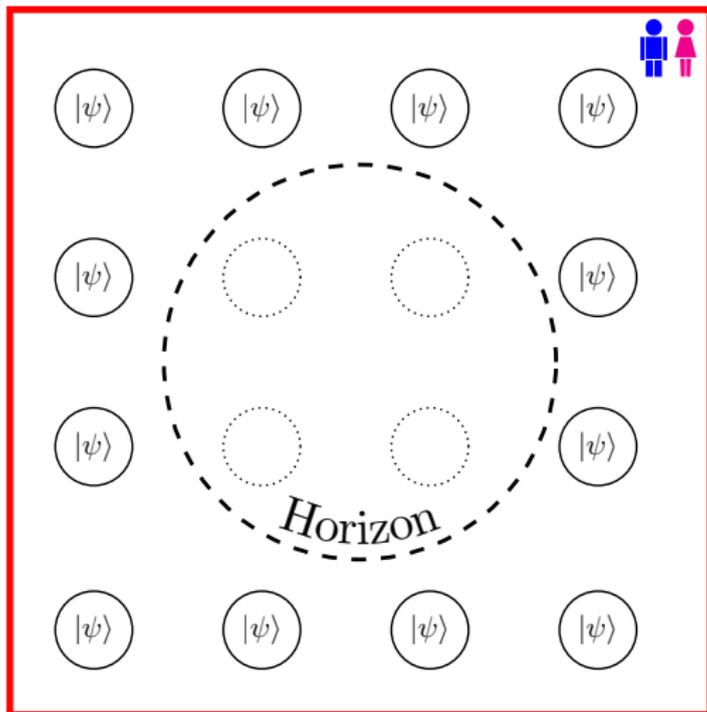
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State of the system:

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Suggested resolutions

- Remnants
- Firewalls
- Fuzzballs
- Supertranslations
- Quantum Gravity

Cosmological implications: PBHs

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- *Primordial* black holes could be **dark matter**
- The smaller a black hole the higher its temperature
 - ⇒ Small black holes evaporate quickly
 - ⇒ Minimum size bounded by the age of the Universe

Important relations to remember

$$T \propto \frac{1}{M} \quad \Rightarrow \quad \text{lifetime} \propto M^3$$

- *PBHs smaller than $\sim 10^{15} \text{g}$ will evaporate too early!*

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- The recent LIGO discovery sparked renewed interest
 - $30 M_{\odot}$ BHs are astrophysically non-trivial

PBHs do not seem a robust candidate for all **DM**

; see for example
[Carr et. al. \(17\)](#)

Vacuum Stability of the Standard Model

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Meet the Higgs Bubble That Will Destroy the Universe. Maybe.

By Erik Vance, Live Science Contributor | April 6, 2018 07:43am ET

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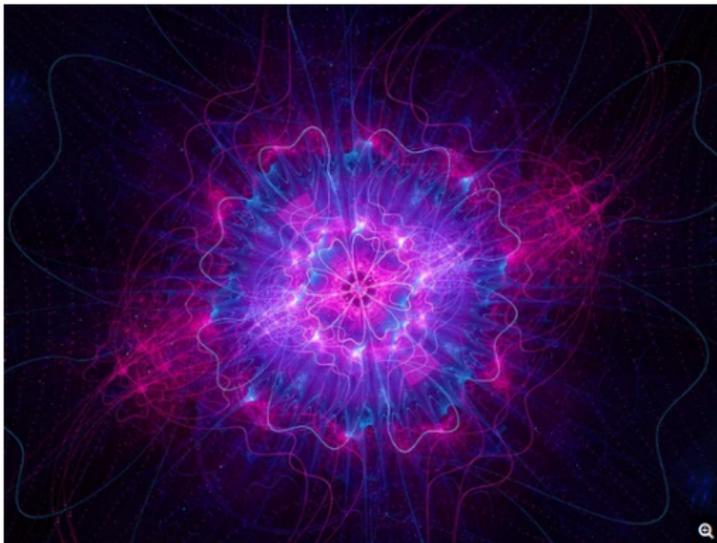
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An artist's conception of the Higgs boson.

Credit: Shutterstock

Scientists say they know how the universe will end. It won't be a cosmic collapse but rather a giant cosmic bubble that devours everything in its path.

Was it Gravity that Saved the Universe After the Big Bang?

By **Matteo Bonetti** - April 5, 2017



One thing that's always puzzled physicists is how the universe didn't collapse immediately after the Big Bang. According to the studies carried out involving the Higgs particle, the production of these during

Motivation

- One of the most profound implications of QFT is the *Renormalization Group*
 - Parameters of the theory **run**

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- **Example:** The Yukawa theory

$$S = \int d^4x \left[\frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{\lambda}{4} \varphi^4 + \bar{\psi} \not{\partial} \psi - g \varphi \bar{\psi} \psi \right].$$

- Classically

$$V(\varphi) = \frac{\lambda}{4} \varphi^4$$

- But in QFT

$$V(\varphi) \approx \frac{\lambda(\varphi)}{4} \varphi^4$$

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- Precise behaviour depends on the initial parameter values

$$\lambda_0 \leq \frac{2g_0^2}{3} \quad \Rightarrow \quad \lambda(\varphi \rightarrow \infty) \rightarrow \pm\infty.$$

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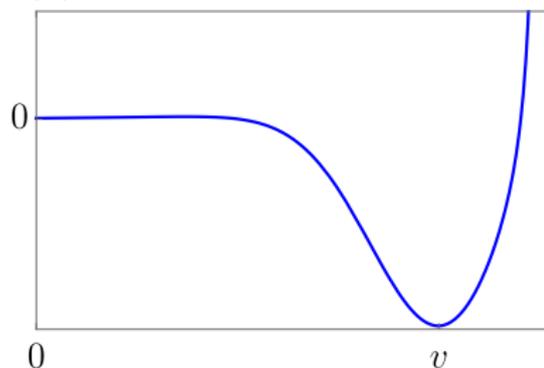
$$\lambda_0 \leq \frac{2g_0^2}{3} \quad \Rightarrow \quad \lambda(\varphi \rightarrow \infty) \rightarrow \pm\infty.$$

In curved space this gets modified!

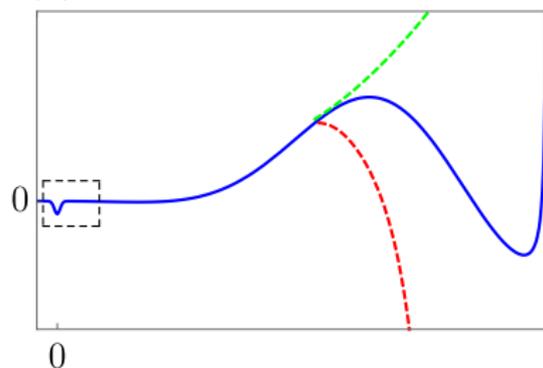
Standard Model Higgs potential

$$V(\phi) \approx -\frac{m^2}{2}\phi^2 + \frac{\lambda(\phi)}{4}\phi^4$$

$V(\phi)$



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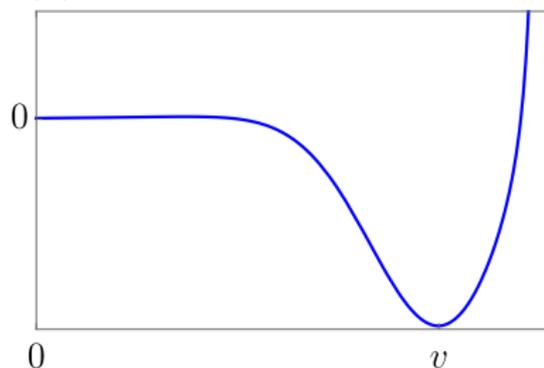


- Minimum at $\phi = v$
- New minimum $\Leftrightarrow \lambda(\phi) < 0$
- A vacuum at $\phi \neq v$ incompatible with observations
- Sensitive to M_h and M_t

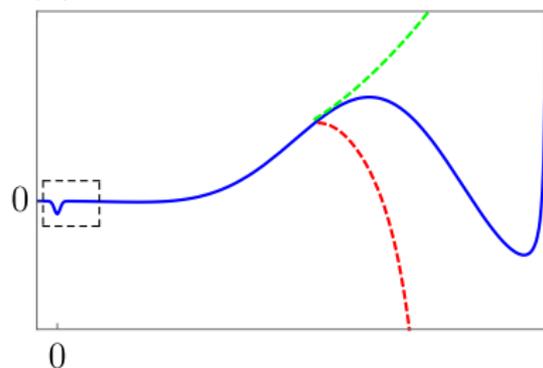
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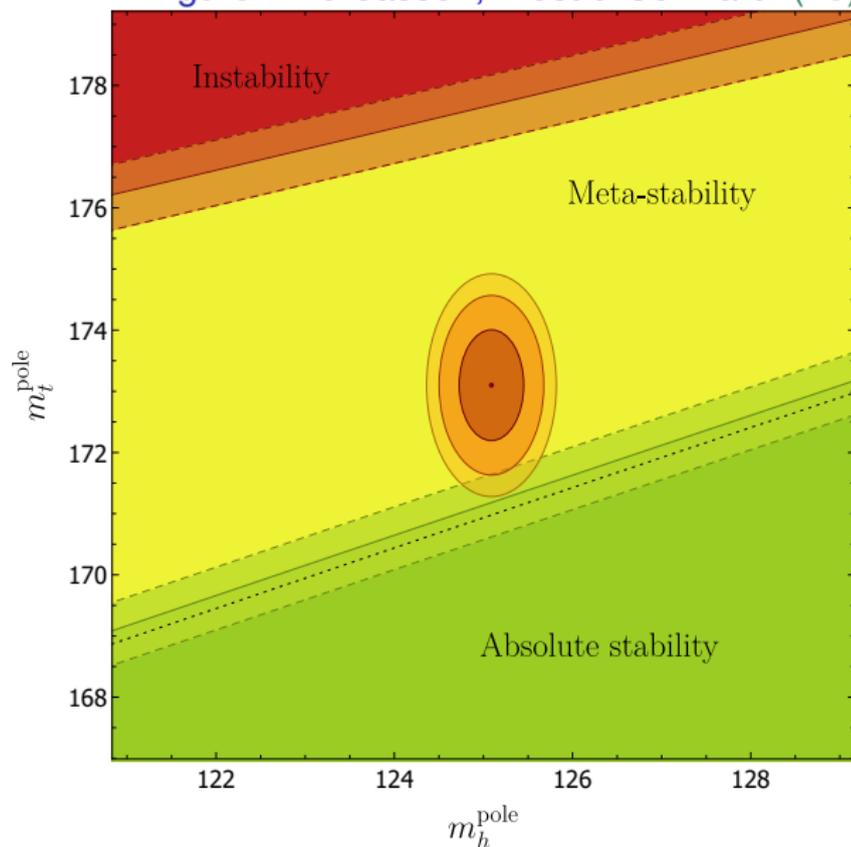
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- Minimum at $\phi = v$
- Sensitive to M_h and M_t
- New minimum $\Leftrightarrow \lambda(\phi) < 0$
- A vacuum at $\phi \neq v$ incompatible with observations
- What about the early Universe (inflation, reheating)?
- New physics needed to stabilize the vacuum?

Current status

Figure: Andreassen, Frost & Schwartz (18):



Fate of the Standard Model

- At the moment metastability is not an issue

Andreassen, Frost & Schwartz (18):

With 95% confidence, we expect our Universe to last more than 10^{58} years.

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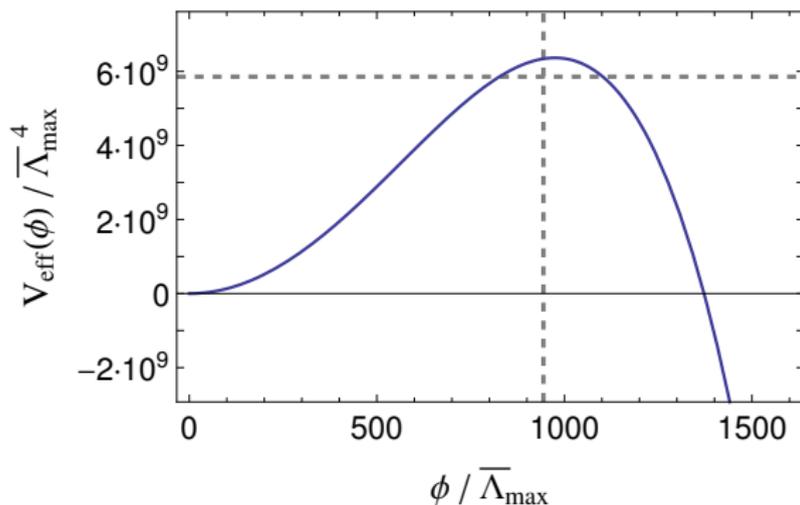
Currently actively studied!

Needs QFT in curved space!

Stabilization from curvature corrections

- The quantum corrected potential

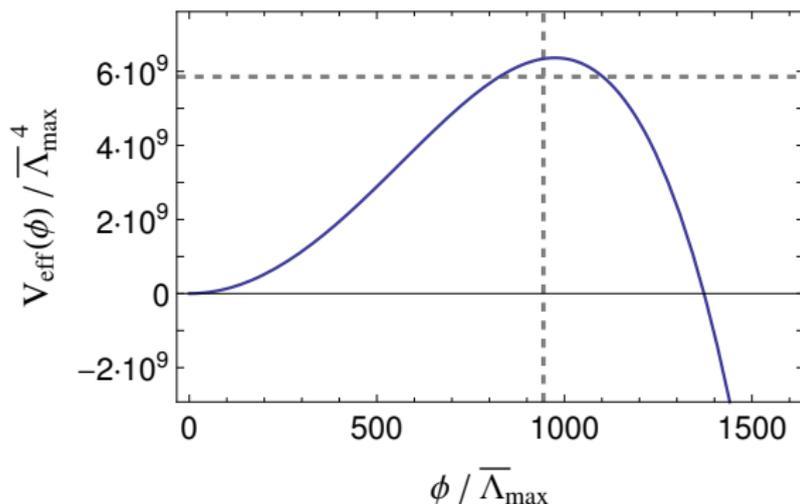
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- Even if $\lambda(\mu) < 0$, the potential can be stable due to ξ
- ξ is *generated* by quantum effects on a curved background
Herranen et al. (14)

Summary

- 1 Introduction
- 2 Cosmological particle creation
- 3 Black hole evaporation
- 4 Electroweak vacuum stability
- 5 Summary**

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QFT in curved spacetime

- A window into quantum gravity
 - However only an approximate approach
- Can be used to show that black holes evaporate
 - From a theoretical point very important
 - Also relevant for cosmological applications
- Crucial for studying the Standard Model in the early universe
 - The electroweak vacuum may be destabilized