Quantum Gravity (= QG)

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(lectures in Finnish, notes in English)

Problem: Our theories of gravity (Newton, Einstein) are classical
The Planck constant does not appear in their predictions, only c, G

Where are the phenomena and what are the laws of nature requiring c,G, $\hbar$?

Where could one find QG effects?

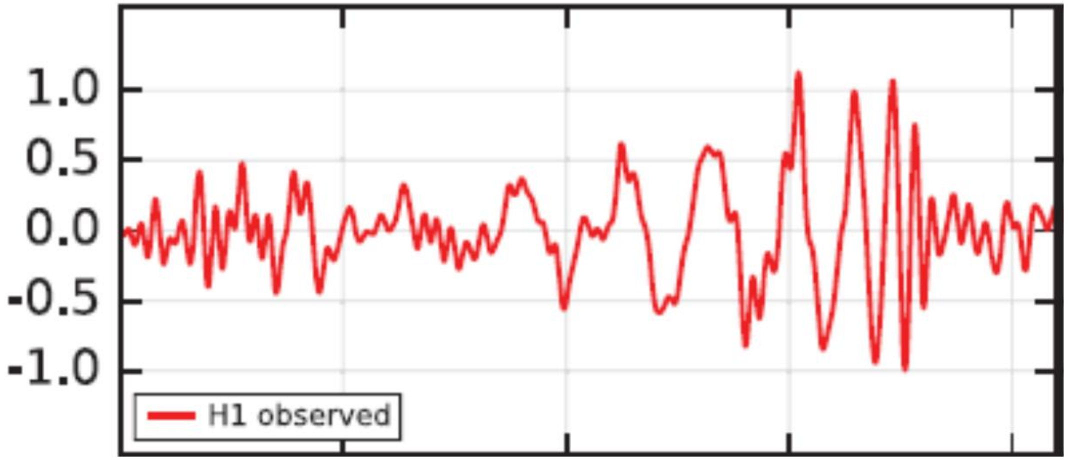
I believe black holes have to be observed and understood

These notes are posted in http://www.helsinki.fi/~kajantie/quantgrav18.pdf

Gravity

as we know today is a classical theory, some coordinates t, x^i, metric tensor $g_{mn}(x)$ solved in terms of matter from Einstein's equations

The famous gravitational wave signal GW150914:



was just oscillations of spatial comps of the deviation of spatial comps of $g_{mn}(x)$ from flat space, computable from

$$(\partial_t^2 - \nabla^2) h_{ij} = 16\pi G\, T_{ij}$$

1. $c,\ \hbar,\ G,\ G\hbar$

c Special relativity SR you need when v/c is non-negligible

$$c = 299792458\,\mathrm{m/s} \quad c^2 \approx 10^{17}$$

$\hbar$ Quantum mechanics QM you need when

$$p \cdot x \sim \hbar = 1.0546 \cdot 10^{-34}\,\mathrm{J\,s} = 6.582 \cdot 10^{-25}\,\mathrm{GeV\,s}$$

G General relativity GR you need for large O(1)

$$\frac{2GM}{c^2 r} \equiv \frac{r_s}{r} \qquad r_s = r_{\mathrm{Schwarzschild}} = 2GM/c^2$$

$$r_s(\mathrm{Earth}) = 0.88\,\mathrm{cm} \quad r_s(\mathrm{Sun}) = 2950\,\mathrm{m}$$

$G\hbar$ Quantum Gravity you need when all consts are involved

Planck units:

$$
\begin{aligned}
\text{mass} &= \sqrt{\hbar c/G} = 2.177 \cdot 10^{-8}\,\text{kg} \\
\text{length} &= \sqrt{\hbar G/c^3} = 1.616 \cdot 10^{-35}\,\text{m} \\
\text{time} &= \sqrt{\hbar G/c^5} = 5.391 \cdot 10^{-44}\,\text{s} \\
\text{power} &= c^5/G = 3.63 \cdot 10^{52}\,\text{W}
\end{aligned}
$$

$$
M_p = \sqrt{\frac{\hbar c}{G}} = 21.8\mu\text{g} = 1.22 \cdot 10^{19}\,\text{GeV}/\text{c}^2 = 543\,\text{kWh}
$$

G is ”small”; ratio of Coulomb and gravitational energies in an H atom:

$$
\frac{V_{\text{grav}}}{V_{\text{ED}}} = \frac{mMG}{\alpha\hbar c} = \frac{1}{\alpha}\,\frac{mM}{M_{\text{Planck}}^2} = \frac{137}{1836}\,\frac{M_{\text{proton}}^2}{M_{\text{Planck}}^2} \sim 10^{-38} \quad \sim \frac{r_{\text{Schw}}^p}{r_{\text{class}}^e}
$$

Maybe there are ”tabletop” exps where this ratio is larger?

2. Reminder: Quantum Mechanics

Mechanics is classical, involves just ordinary commuting numbers:

$$F = m\,\ddot{x}, \qquad E = \tfrac{1}{2}mv^2 + V(x), \qquad \mathbf{L} = \mathbf{r} \times \mathbf{p}, \qquad rp = pr$$

Works perfectly until one is able to do experiments at scales for which

$$p \cdot x \sim \hbar = 1.0546 \cdot 10^{-34}\,\mathrm{J\,s} = 6.582 \cdot 10^{-25}\,\mathrm{GeV\,s}$$

Crucial properties of QM are discrete energy levels $E=E_n$ and uncertainty principle:

$$\Delta p\,\Delta x \geq \tfrac{1}{2}\hbar$$

One neede experimental guidance to get here! Also for gravity!

3. Special relativity and QM coexist as relativistic quantum field theory:

The standard model works up to energies around 100 GeV

What is there between 100 GeV and 10^{19} GeV, QG?

Desert between 100 and 10^{19} GeV, with lots of life: dark matter, DB!??

Something drastic happens to coordinates at Planck scales:

Try to localize a massless (or $m \ll M_{planck}$) particle to better than Planck length

Then its momentum uncertainty is $> hbar/l_{planck} = M_{planck}\, c$

Effectively you squeeze an energy $M_{planck}\, c^2$ into l_{planck}: becomes a black hole!

The expectation is that our space-time with t,x as coordinates emerges from QG!

No quantal effects have been observed:
Discrete states
Double slit experiment
Interference, entanglement
Uncertainty relations

Theoretically, one can take Einstein, find out coordinates q_i and momenta p_i and the Hamiltonian and try to quantize as mechanics – dead end (Loop quantum gravity)

String theory is quantum mechanics containing a spin 2 massless graviton but has supersymmetry and lives in 10-dimensional space – so far dead end, too

Chief problem: spacetime must be an emergent quantity, too!

4. Einstein classical gravity, black hole solution

$v_{escape}^2 = 2GM/r = c^2$ if $r = r_{Schw} = 2GM/c^2$

Einstein's vacuum equations $R_{\mu\nu} = 0$ for the metric tensor $g_{mn}(x)$ in the coordinates $x^\mu = (t, r, \theta, \phi)$ have the Schwarzschild solution

$$g_{\mu\nu}(x) = \begin{pmatrix} -(1-\frac{r_s}{r}) & 0 & 0 & 0 \\ 0 & \frac{1}{1-r_s/r} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix}$$

$$d\tau^2 = (1-\frac{r_s}{r})dt^2 - \frac{1}{1-r_s/r}dr^2 - r^2\, d\Omega^2$$

Black hole with horizon at $r = r_s$

The only ”curvature singularity” is at r=0 but this singularity is hidden beyond the ”event horizon” at $r = r_s$

You do not understand QG until black holes are understood

So far just a classical solution, no hbar. This appears with Hawking:

5. Hawking radiation

BHs have temperature $T_H = \frac{\hbar c}{4\pi r_s} \sim \frac{10^{23}\,\mathrm{kg}}{M}\,\mathrm{K}$ entropy $S = \frac{c^3}{\hbar}\frac{A}{4G} \equiv \frac{A}{4L_P^2}$

BH thermo: $E = Mc^2 \quad dE = T_H\, dS$ Carlip 1410.1486

BHs are unstable $\frac{dM}{dt} \sim -r_s^2 T_H^4 = -\frac{\hbar c^4}{60\cdot 265\pi}\frac{1}{G^2M^2}$ $M(t) = M_0(1-\frac{t}{t_{\rm ev}})^{1/3}$

Here decay to photons

$$t_{\rm ev} = 20\cdot 256\pi\frac{M_0^3}{M_p^3}\,t_p \sim 10^{-16}s\,\frac{M_0^3}{\mathrm{kg}^3}$$

Milky Way BH is BIG: $M \sim 4\cdot 10^6\,M_{\rm sun} \quad r_s = 12\cdot 10^6\,\mathrm{km} \approx \frac{1}{10}\,\mathrm{au}$

Very active research topic, information paradox, firewalls,.... Harlow 1409.1231

Questions:

What really happens when you fall through the horizon? Classical gravity tells: nothing. According to equivalence principle you just fall through

What are the microstates of BH entropy?

Information paradox: BH radiates, decreases its mass until Mplanck,
How does it disappear? Where is the info of everything fallen into BH gone?

Key questions for QG!!

Literature

Sean Carroll, Spacetime and Geometry, Addison Wesley 2004

Steven Weinberg. Gravitation and Cosmology, John Wiley, 1972

N. D. Birrell and P. C. W. Davies, Quantum fields in curved space, Cambridge University Press, 1999

Robert M. Wald, General Relativity, U of Chicago Press 1984

Carlo Rovelli, Quantum Gravity, Cambridge U Press, 2004

http://math.ucr.edu/home/baez/hamiltonian/hamiltonian.html

Blogs

To be updated on QG it pays absolutely to follow certain physics blogs:

http://backreaction.blogspot.fi/ has a large number of articles on QG, search for them!

http://resonaances.blogspot.fi/ Particularly high quality, authoritative
19 Apr 18 discusses graviton mass

http://www.math.columbia.edu/~woit/wordpress/

Everything worth knowing is linked here

http://www.science20.com/quantum_diaries_survivor Exp part physics

https://www.quantamagazine.org/physicists-find-a-way-to-see-the-grin-of-quantum-gravity-20180306/ A very readable article on tabletop experiments for QG

6. Conclusions

This was an exceedingly brief discussion of Quantum Gravity

One has the feeling that the final theory of QG is still very far. Too little help from observation!

But it is there: go and find it!