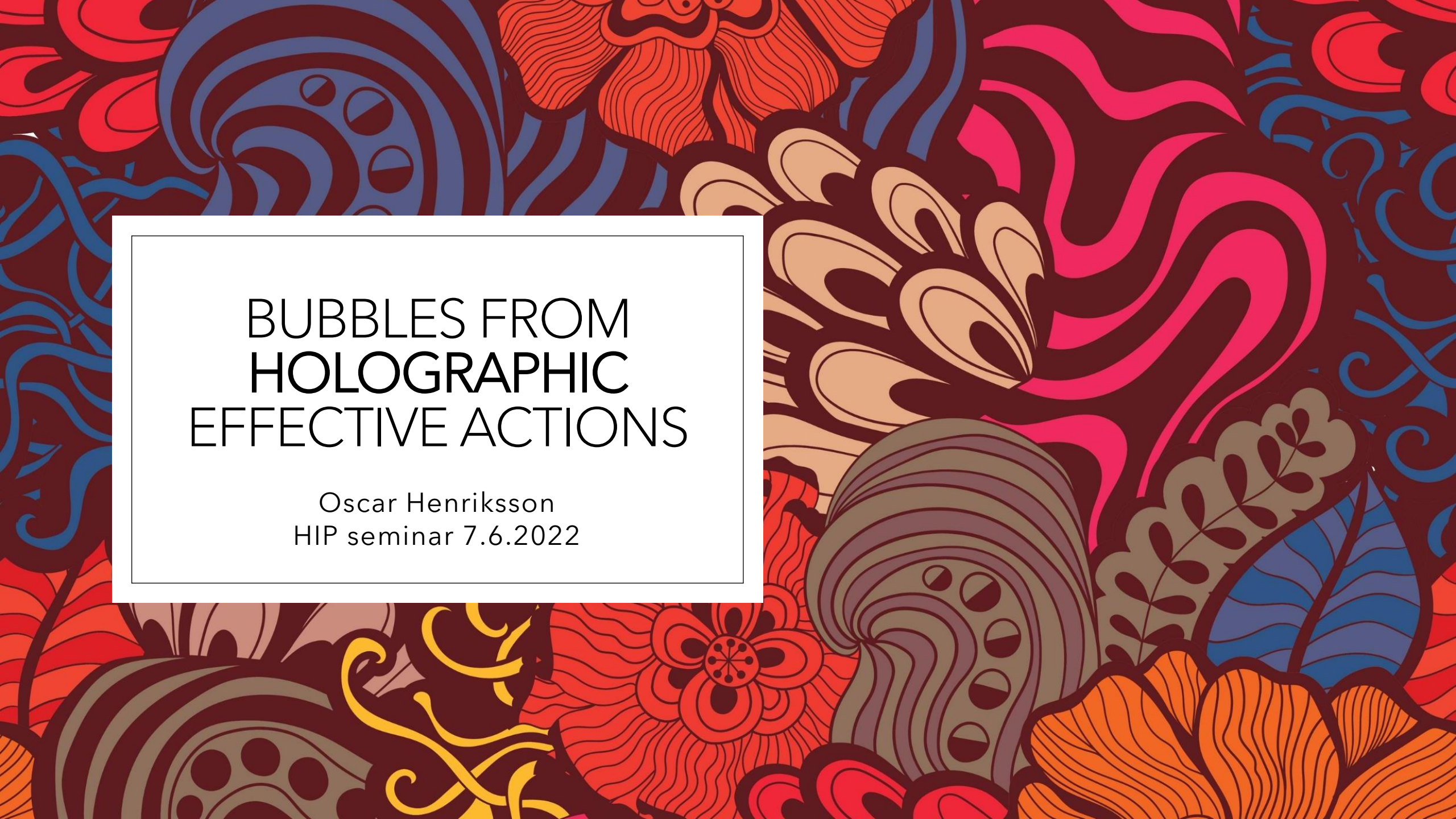
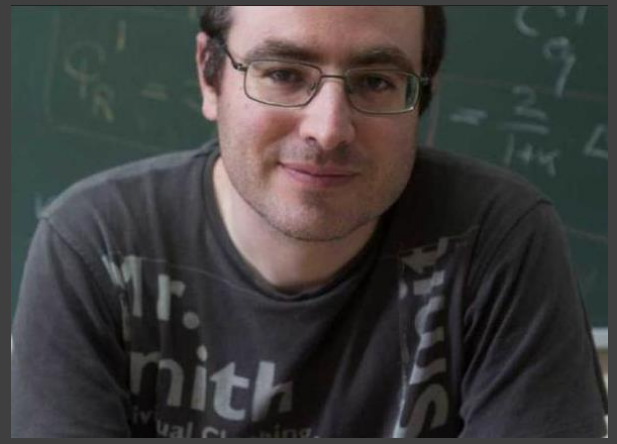


BUBBLES FROM **HOLOGRAPHIC** EFFECTIVE ACTIONS

Oscar Henriksson

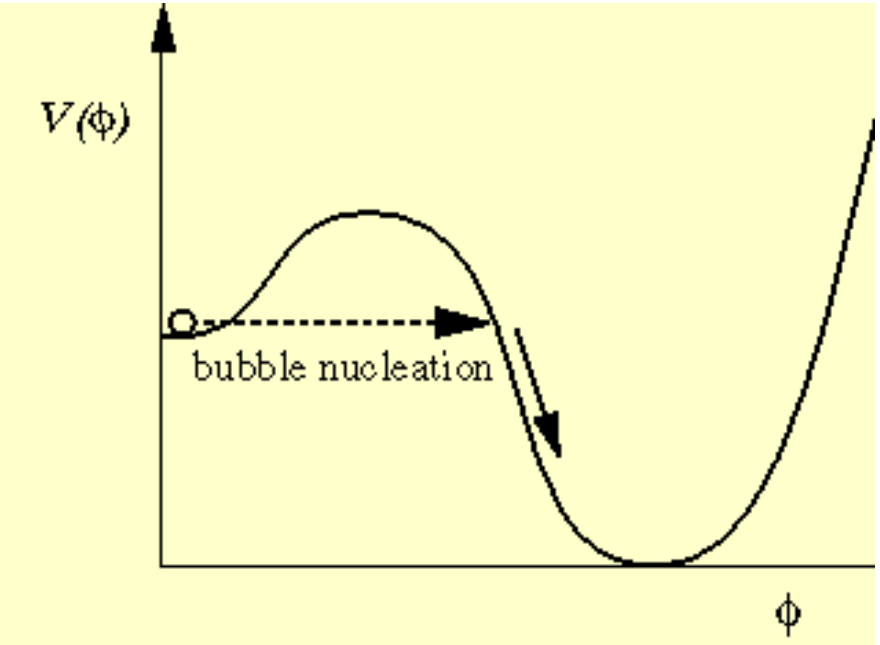
HIP seminar 7.6.2022



With Fëanor Reuben Ares, Mark Hindmarsh, Carlos Hoyos & Niko Jokela

- ”Effective actions and bubble nucleation from holography”, 2109.13784
- ”Gravitational Waves at Strong Coupling from an Effective Action”, 2110.14442

...as seen in the papers!



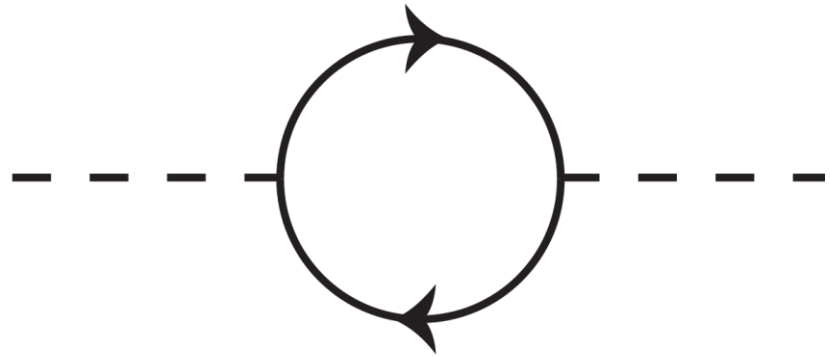
Phase transitions are all around us!

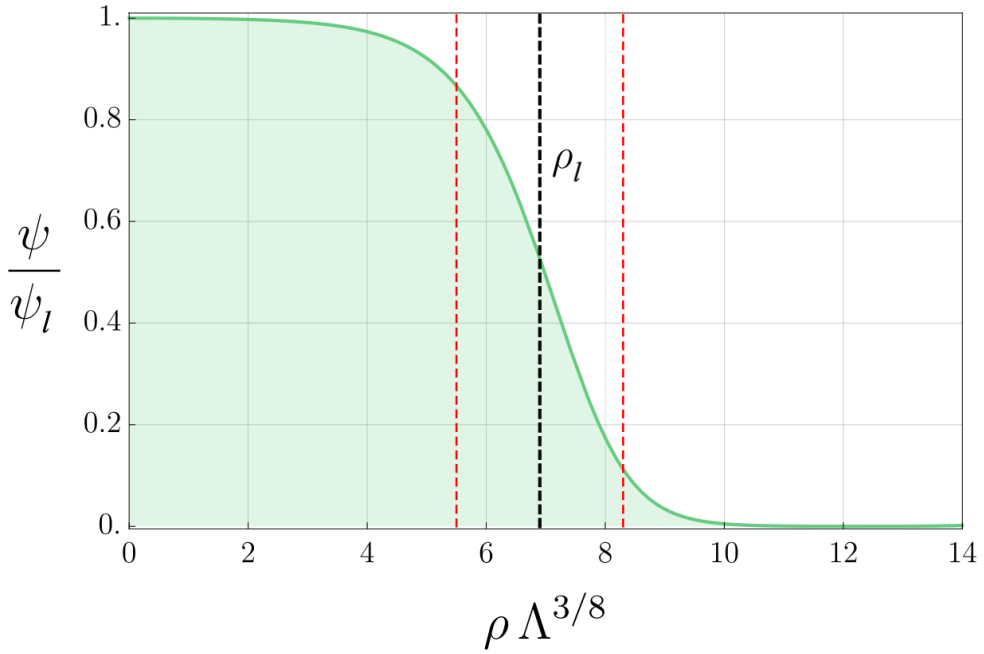
1st order transitions typically occur through **bubble nucleation**

Quantum fields are all around us!

- In QFT, bubble nucleation can be studied “semi-classically” by finding (Euclidean) bubble solutions
- Quantum corrections can be important → instead of classical action, use **quantum effective action (EA)**

$$\Gamma[\Psi] = \mathrm{W}[\mathrm{J}] - \int d^4x\, \Psi \mathrm{J}$$





From phase transitions to gravitational waves

- With physics **beyond the standard model**, 1st order transitions *might* have happened in the early universe…
- Collisions of such bubbles produce gravitational waves → detectable by e.g. LISA?
- Important quantities: Nucleation temperature T_n, transition strength α, transition rate β, wall speed v_w, …
- Normally computed using perturbation theory, assuming weak coupling

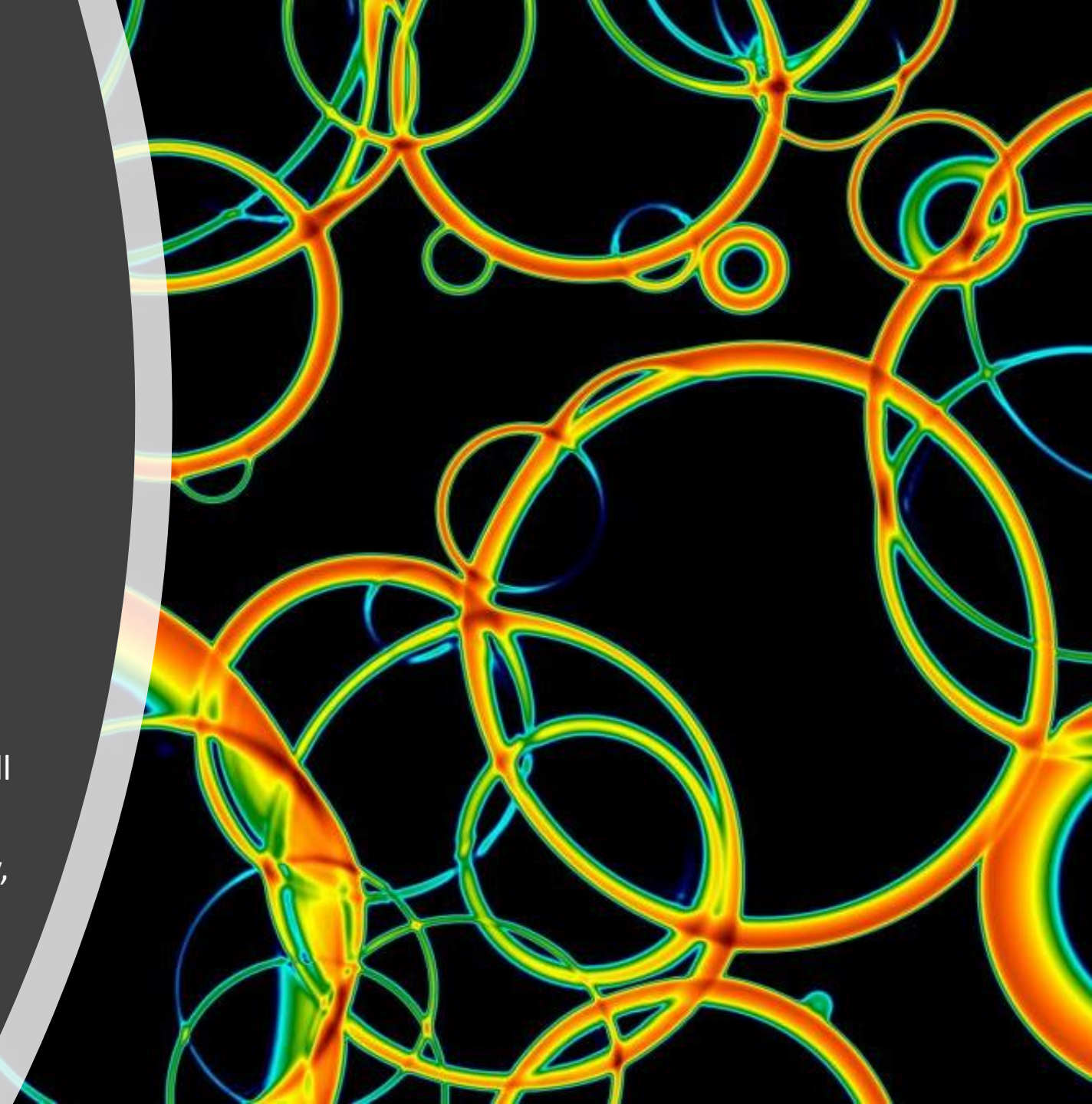
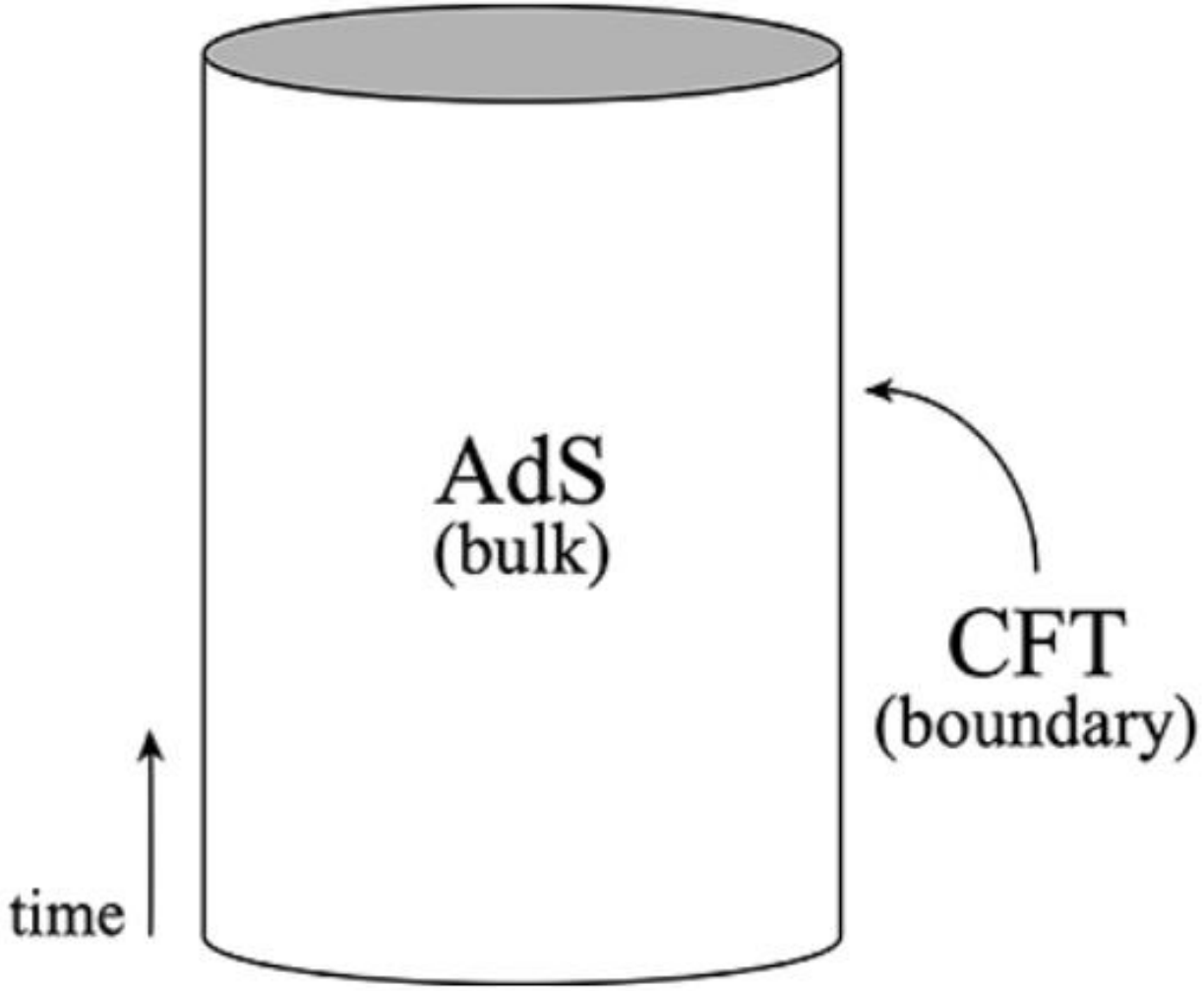


Figure by David Weir

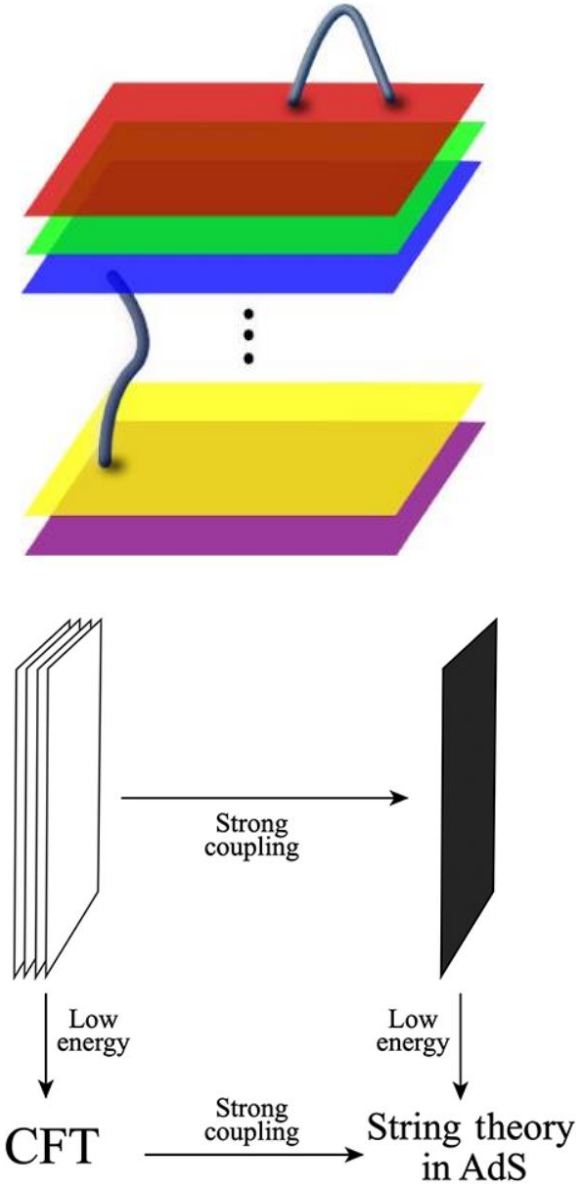
At strong coupling, life is hard!

…unless you do holography ;p



Holographic duality: The origin story

- D-branes: String theory objects on which open strings can end
- Start from stack of *N* D-branes at weak coupling
- Low energy: world-volume conformal SU(N) gauge theory (CFT)
- Strong coupling: Gravity description as **“black brane”**
- Near-horizon limit gives string theory in “anti-de Sitter” (AdS) space
- Identifying resulting theories → holographic duality



Figures from Rosen (2009) and Polchinski, Almheiri (2022)

Holographic duality: Some intuition…

- CFT “lives” on boundary of AdS
- Extra dimension in AdS ~ energy scale in dual field theory

USEFUL! Large-N, strong coupling limit in QFT ~ classical gravity limit in string theory

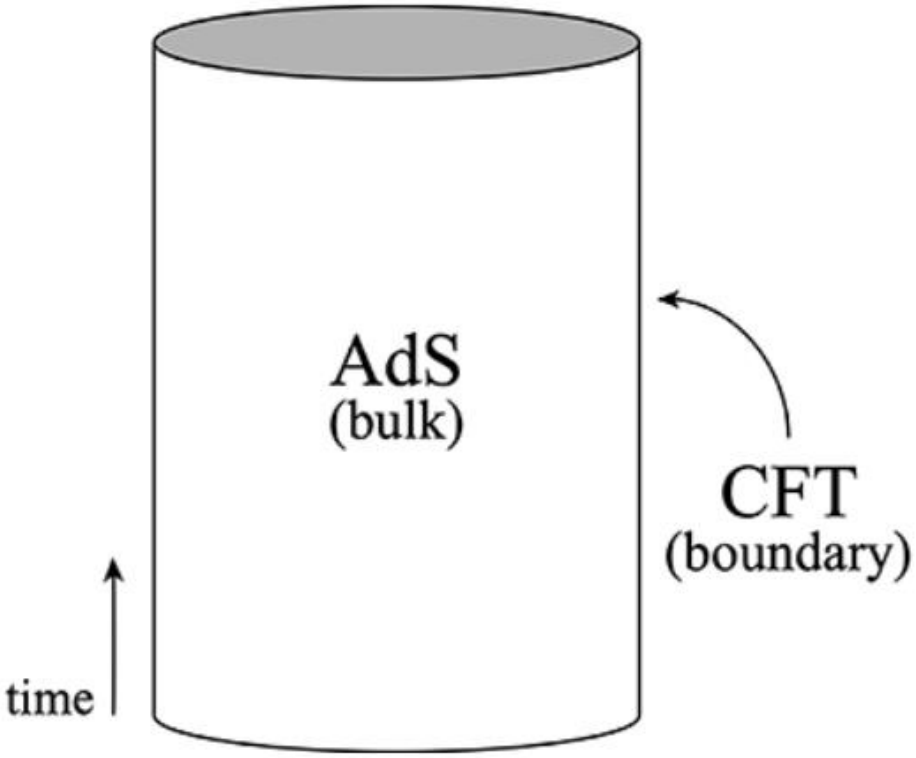


Fig. from Polchinski, Almheiri (2022)

Holographic duality: Dictionary

Quantum field theory	String/gravity theory
Partition function!	Partition function?
“Large N”, strong coupling	Classical gravity
Operators (some, at least)	Fields
Sources	Boundary conditions
Generating function	On-shell gravity action
…	…

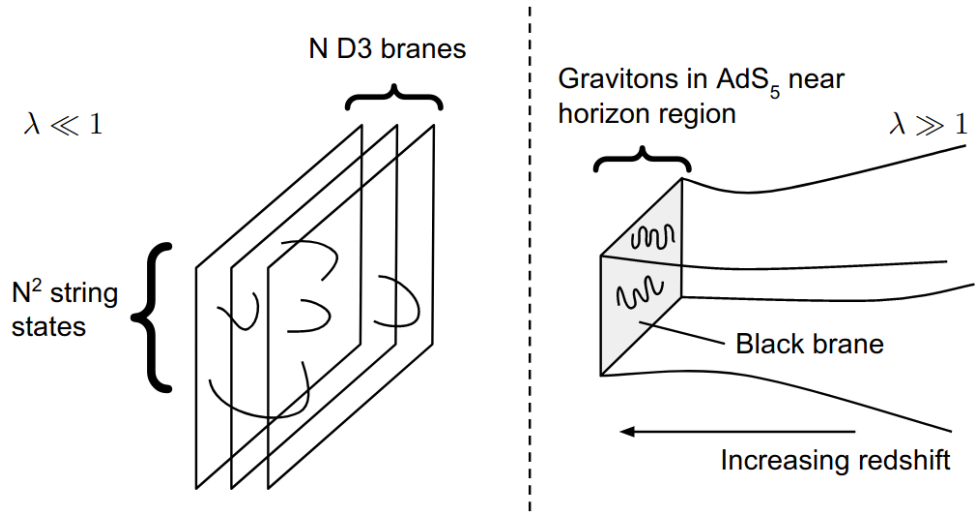


Fig. from Hartnoll et al. (2016)

Simple example… of a **general** approach

We compute the effective action, in a derivative expansion…

$$\Gamma[\Psi] = -N^2 \int d^4x \left\{ V(\Psi) + \frac{1}{2} Z(\Psi)(\nabla\Psi)^2 + \cdots \right\}$$

…using holography.

Our model: A simple gravity-scalar theory

$$S = \frac{N^2}{2} \int \mathrm{d}^4x \sqrt{-g} \left\{ R - \left(\partial_\mu \phi\right)^2 + \frac{12}{L^2} - m^2 \phi^2 \right\}$$

Scalar mass m sets dimension of dual operator Ψ – choose such that **dimension is 4/3**

Multi-trace deformations

The QFT operator Ψ is a **dimension-4/3 scalar**. In large-N lingo, “single trace”.

We can deform the original CFT by “multi-trace” operators:

$$S_{CFT} \to S_{CFT} + \int d^4x \left\{ \Lambda\Psi + \frac{f}{2}\Psi^2 + \frac{g}{3}\Psi^3 \right\}$$

Nice for two (related) reasons:

- Easy in holography: changing boundary conditions in AdS
- Easy in field theory: Give simple and straightforward contributions to the effective action (large-N effect):

$$\Gamma[\Psi] \to \Gamma[\Psi] + \Lambda\Psi + \frac{f}{2}\Psi^2 + \frac{g}{3}\Psi^3$$

Simplest holographic model of a phase transition?

Dim-4/3 model – effective potential

EA is a Legendre transform

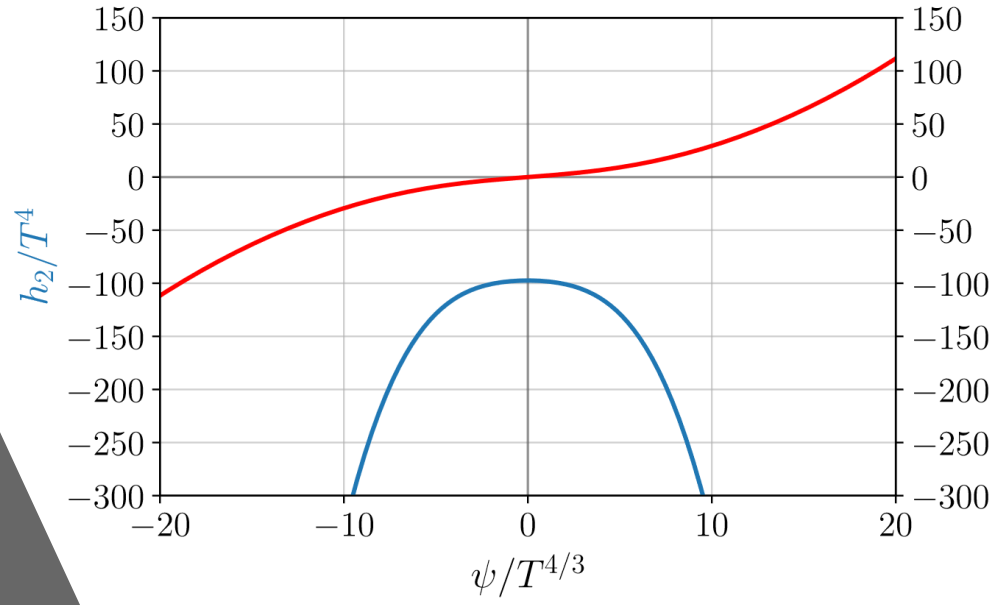
$$\Gamma[\Psi] = \mathrm{W}[\mathrm{J}] - \int d^4x\, \Psi \mathrm{J}$$

Effective potential $V(\Psi)$ is homogeneous limit of EA

→ find exhaustive set of **homogeneous** solutions to gravity equations

Simple ansatz (only radial dependence etc.) reduces Einstein-scalar equations to three coupled ODEs – solve numerically

Result: 1-parameter family of solutions



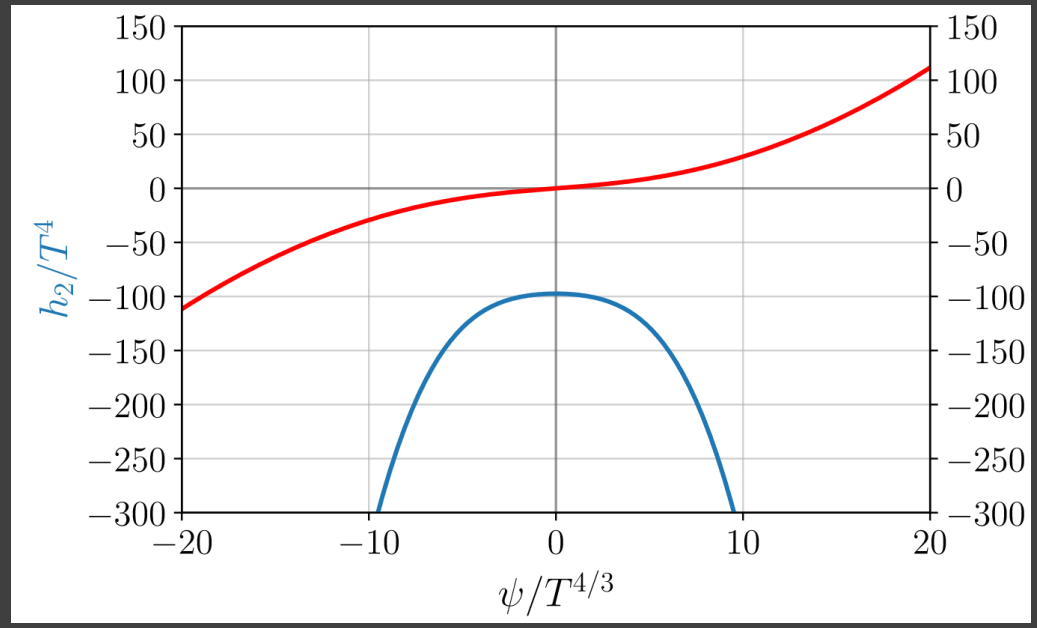
Dim-4/3 model – effective potential

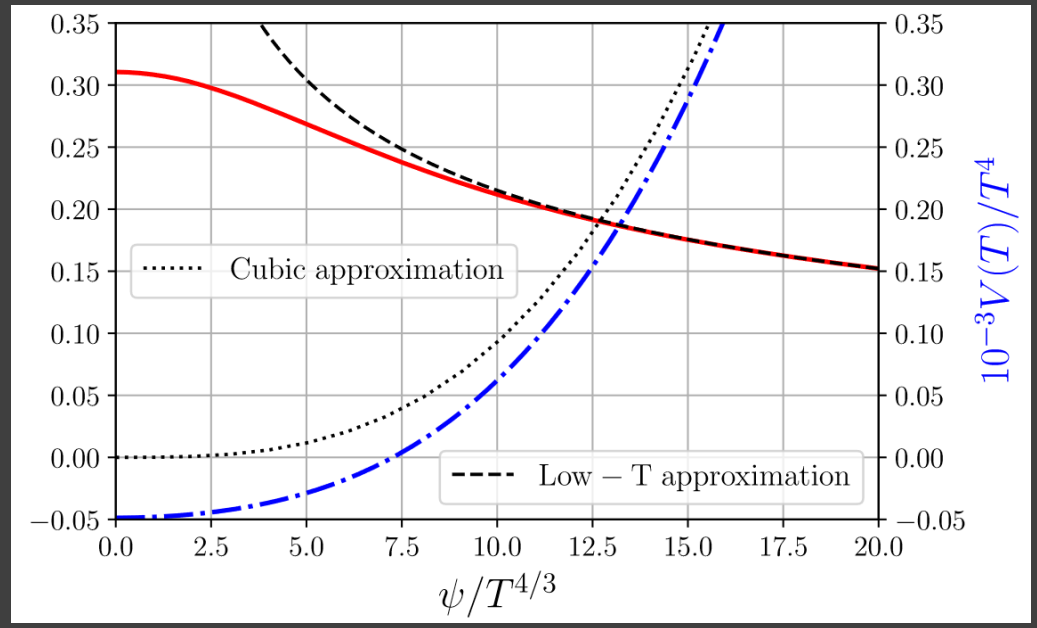
Properties of solutions near AdS boundary give field theory parameters (e.g. sources and expectation values)

Can use to construct effective potential; easiest way is to integrate

$$\frac{dV(\Psi)}{d\Psi} = J(\Psi)$$

Result: Boring! No phase transitions!

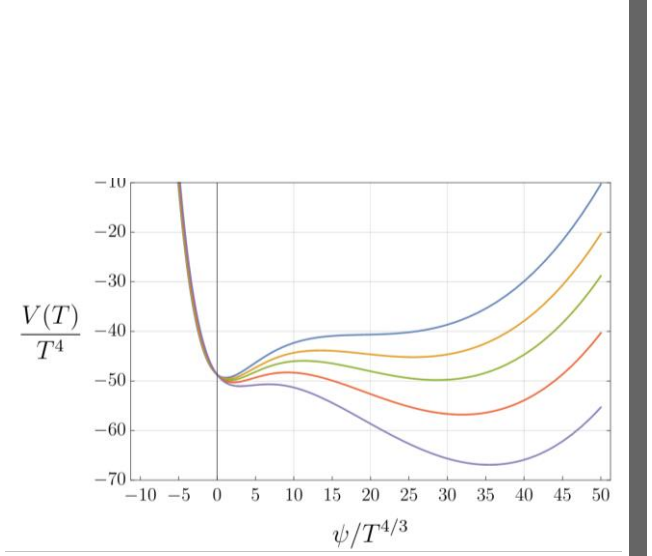


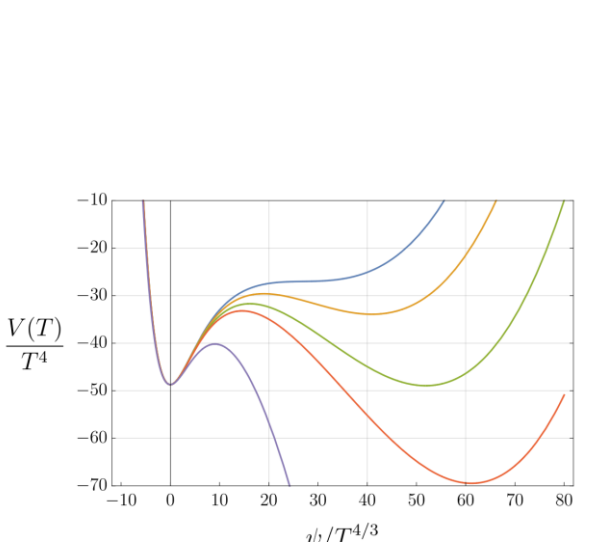


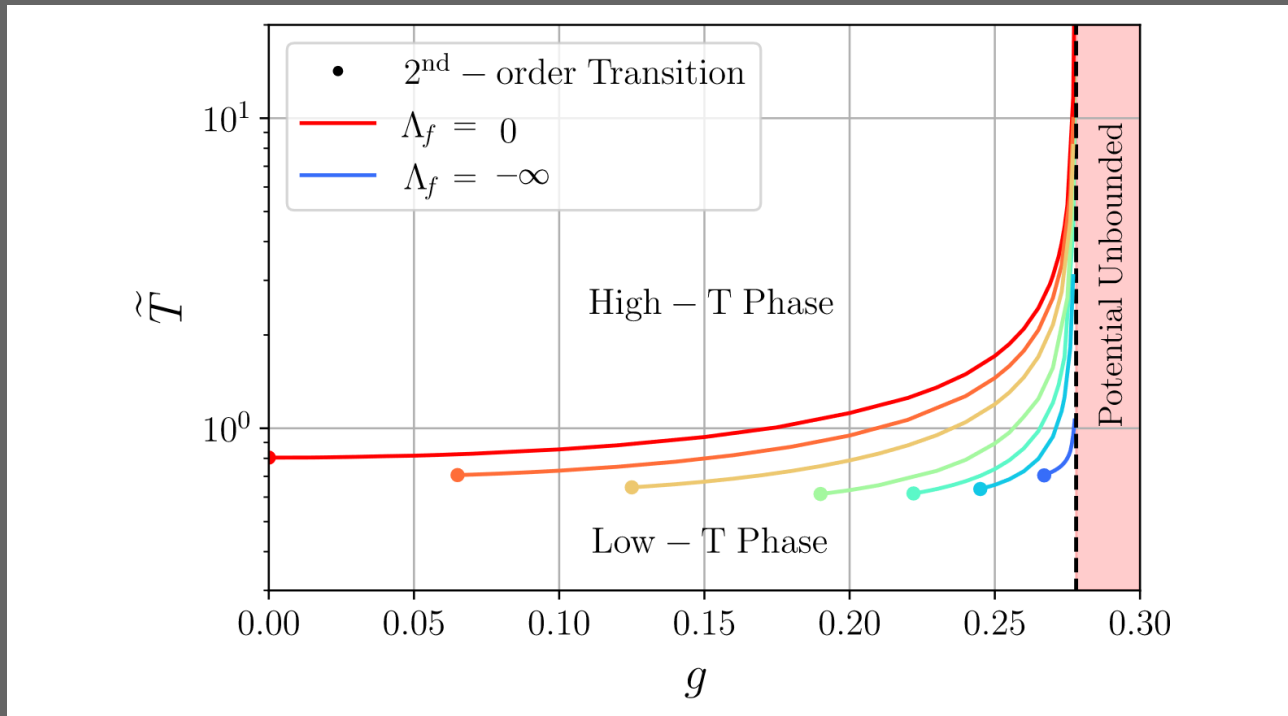
But… by tuning Λ, f, g we can then modify potential to induce phase transitions!

g is important to introduce *non-concavity*

Note: All strong coupling physics is, in some sense, in the undeformed potential

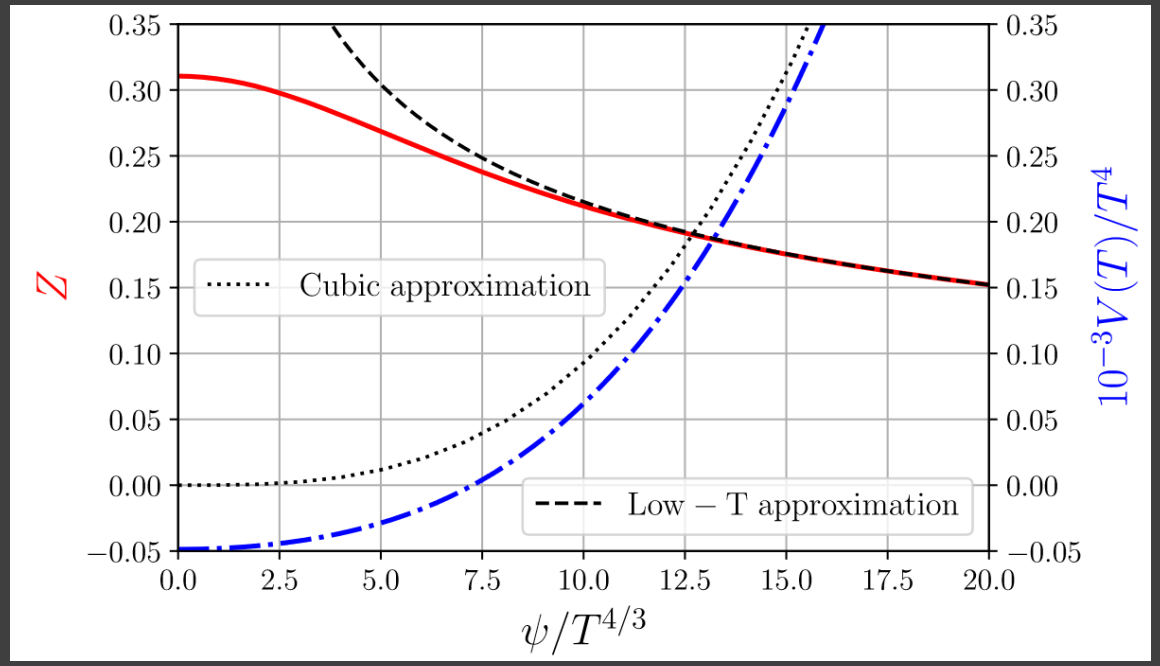






Dim-4/3 model – kinetic term

- Ψ^n part of effective action equals 1PI n-point function
- Derivative expansion ↔ low-momentum expansion
- k^2-part of correlator gives kinetic term!
- In holography, compute two-point functions in a particular QFT state by solving Einstein + scalar equations ***linearized*** around dual homogeneous solution → two coupled linear ODEs, solve again by numerical shooting



Higher order terms in the effective action?

$$\Gamma[\Psi] = -N^2 \int d^4x \left\{ V(\Psi) + \frac{1}{2} Z(\Psi)(\nabla\Psi)^2 + \cdots \right\}$$

Ignore!

…can (and did) compute subset of them by computing two-point function to higher order in momentum.

→ verified that $k^4\Psi^2$-term computed in this way was negligible for our bubble solutions

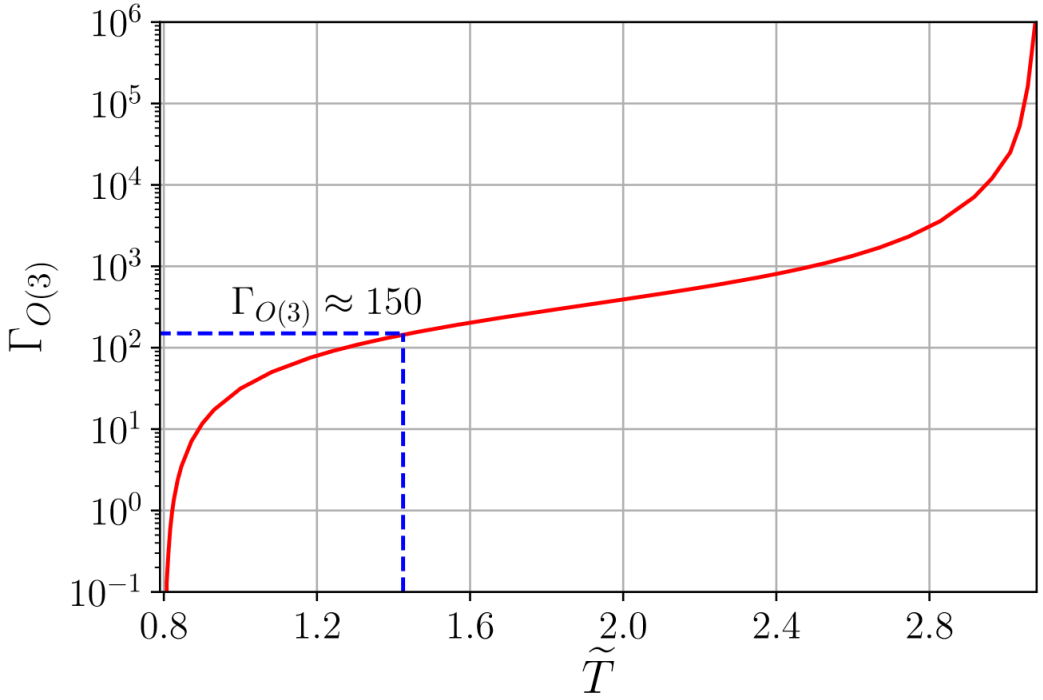
Nucleation at large N – general remarks

Holography useful ↔ large N (“many degrees of freedom”)

→ action proportional to N^2

→ large-N is *semi-classical limit*, nucleation suppressed

In early-universe phase transitions, nucleation happens when action equals 150…



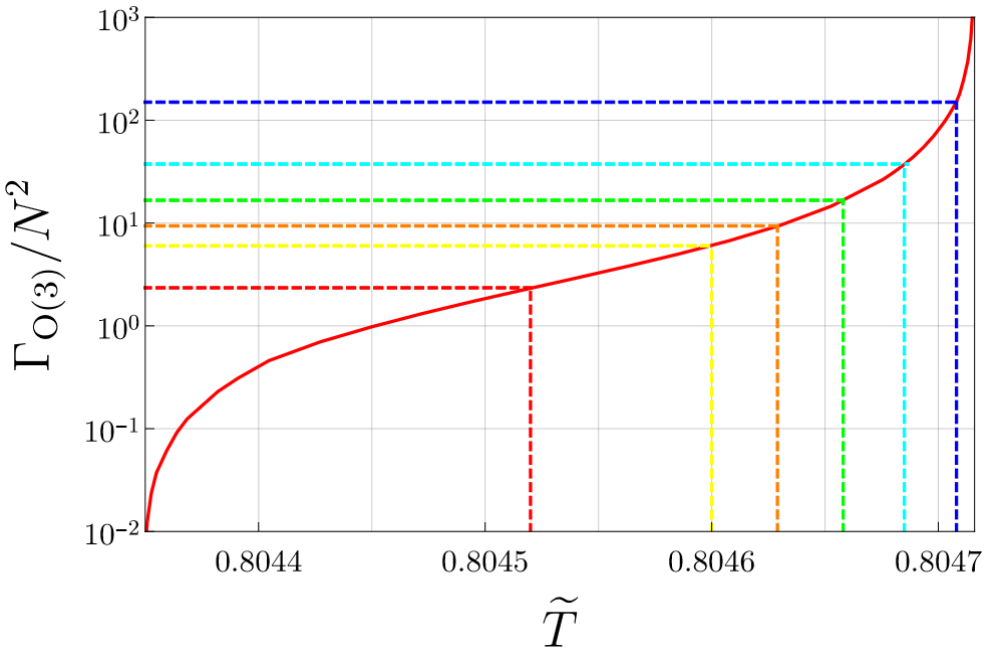
Nucleation at large N – general remarks

Increasing N pushes nucleation temperature down…

In large supercooling limit, can approximate bubble action as $\Gamma \sim N^2(T - T_0)^x$, $x > 0$

This leads to

$$\beta/H_n \sim N^{2/x}$$

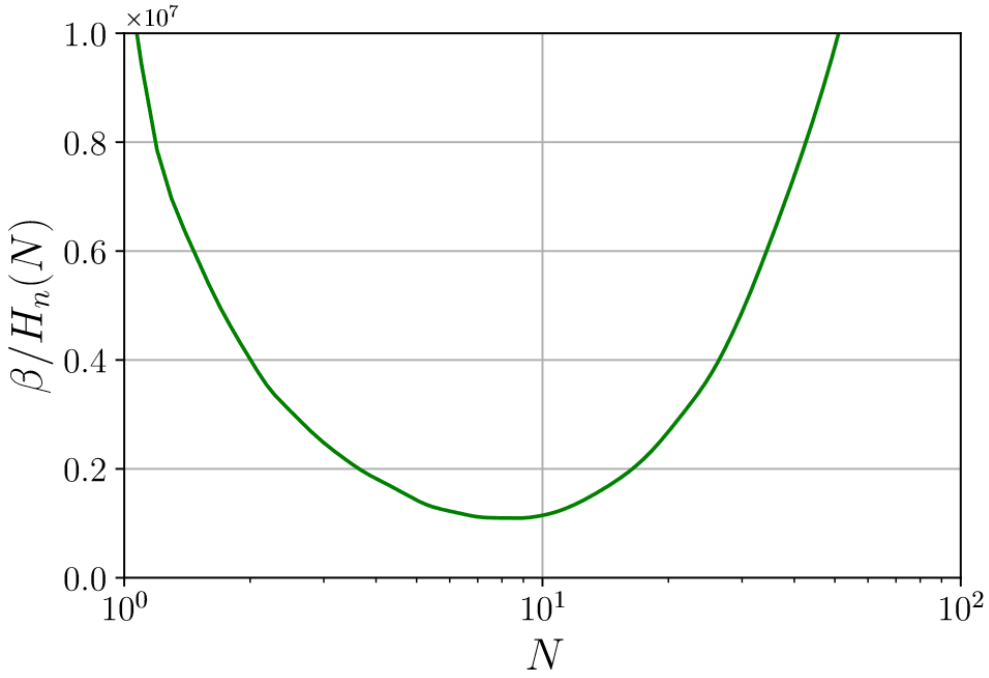


Nucleation at large N – general remarks

However, for finite N, nucleation can also take place for small supercooling (thin wall limit)

Then, $\Gamma \sim N^2(T - T_C)^{-2}$, leading to $\beta/H_n \sim N^{-1}$

Assuming large-N limit is throughout, can have an **optimal N** that minimizes β, leading to larger gravitational wave signals.



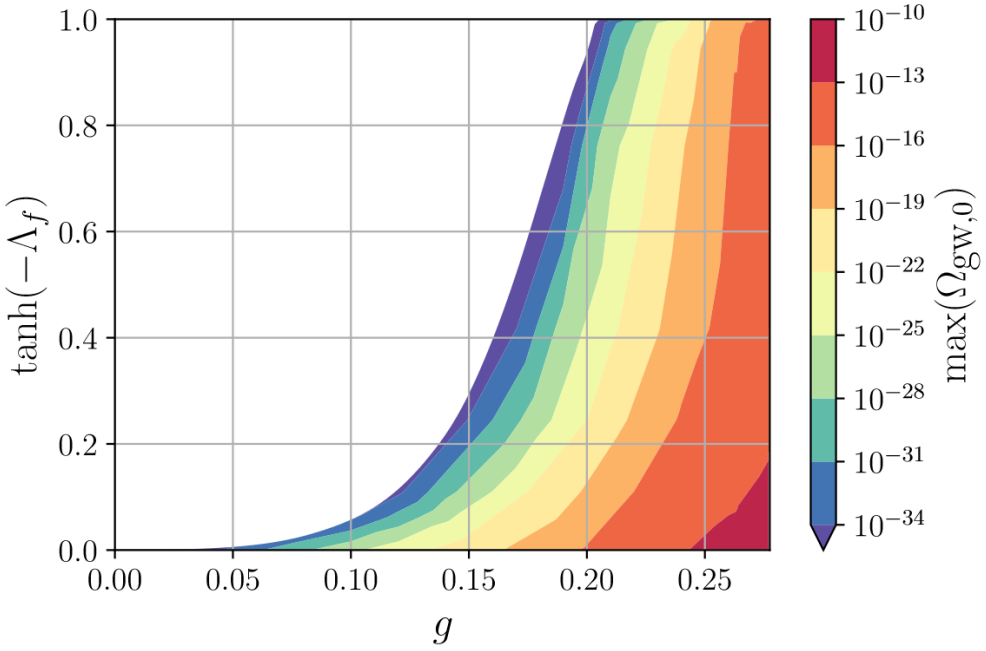
Gravitational waves!

Choosing $N = 8$ (optimal) we compute all quasi-equilibrium GW parameters

Use phenomelogical relationship (from other holographic works!) to estimate wall speed

Use (improved) LISA cosmology working group model to find GW power spectrum →

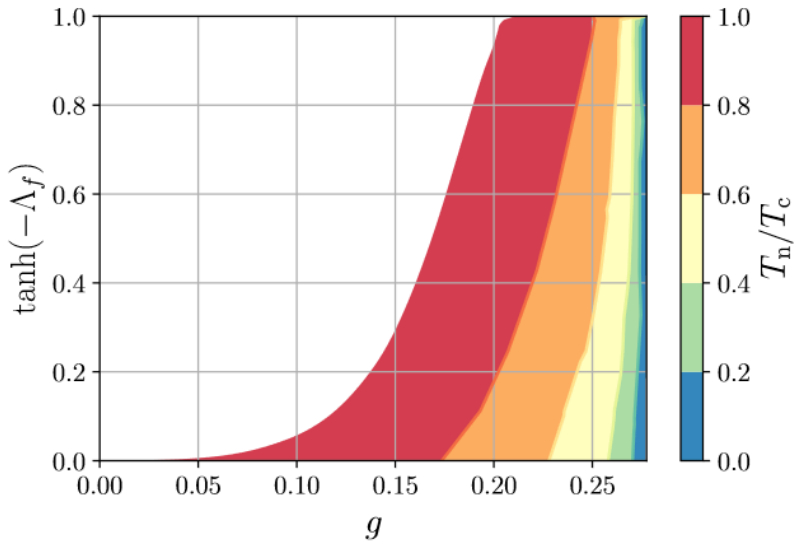
Detectable signal for small Λ and “large” g

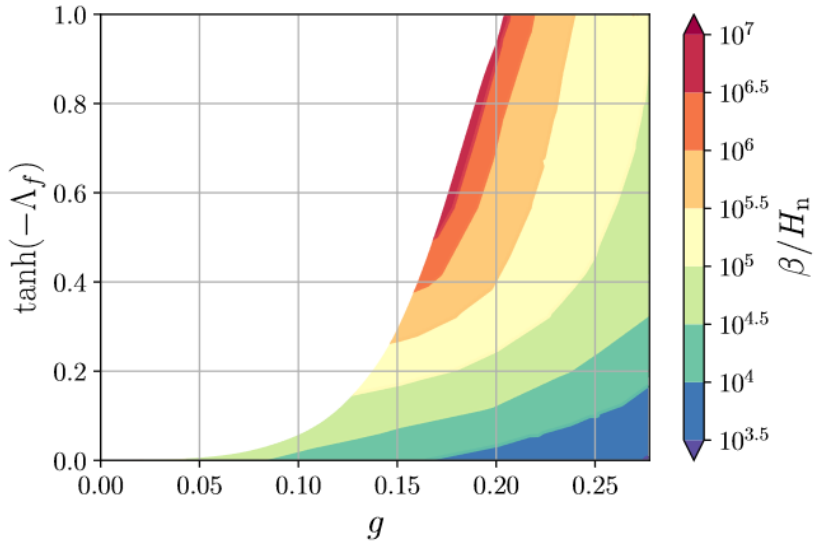


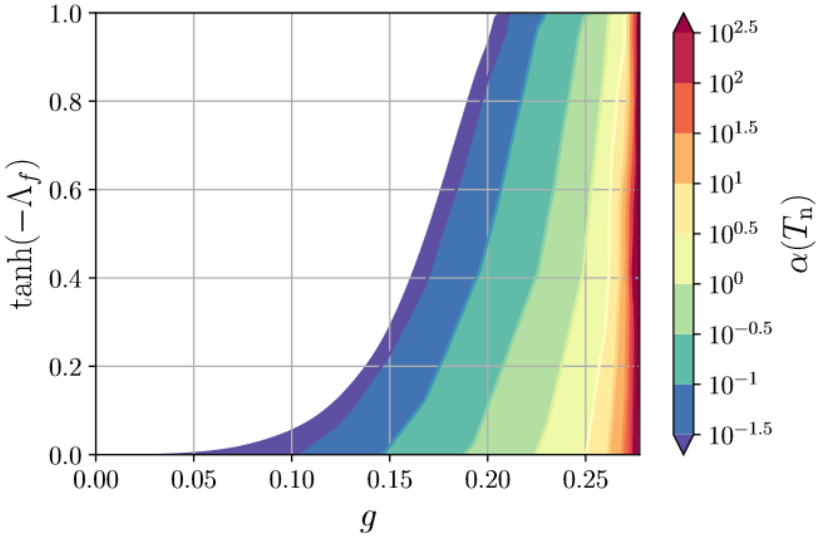
What to **TAKE AWAY**

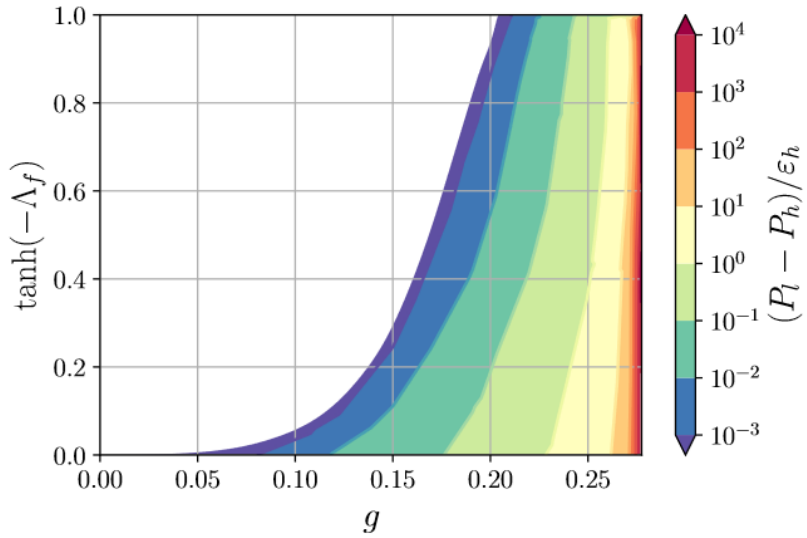
- With holographic duality, life is good at strong coupling!
- Can compute effective action & study phase structure + transitions from homogeneous gravity solutions (“simple”)
- **Straightforward and general** approach to study bubble nucleation at strong coupling, compute parameters relevant for gravitational wave production.

- Future:
  - Wall speeds!
  - More realistic models!
  - Bubble directly in gravity!
  - Finite density!
  - Defects, striped phases etc. from effective actions!
  - Think more about effective actions!


1.0
0.8
0.6
0.4
0.2
0.0
tanh(−Λf)
0.00
0.05
0.10
0.15
0.20
0.25
g
1.0
0.8
0.6
0.4
0.2
0.0
Tn/Tc


1.0
0.8
0.6
0.4
0.2
0.0
tanh(−Λf)
0.00
0.05
0.10
0.15
0.20
0.25
g
10^7
10^6.5
10^6
10^5.5
10^5
10^4.5
10^4
10^3.5
β/Hn


1.0
0.8
0.6
0.4
0.2
0.0
tanh(−Λf)
0.00
0.05
0.10
0.15
0.20
0.25
g
10^2.5
10^2
10^1.5
10^1
10^0.5
10^0
10^−0.5
10^−1
10^−1.5
α(Tn)


1.0
0.8
0.6
0.4
0.2
0.0
tanh(−Λf)
0.00
0.05
0.10
0.15
0.20
0.25
g
10^4
10^3
10^2
10^1
10^0
10^−1
10^−2
10^−3
(Pl − Ph)/εh

Outline

- Phases and bubbles in QFT
- Holographic duality
- Effective action from holography
- A simple “multi-trace” model
- Remarks – bubble nucleation at large N
- Some results