First order phase transitions and holography

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Phase transitions are all around us!

…*might* have happened in the early universe!

- Many BSM proposals lead to 1st order PTs
- Collision of bubbles sources gravitational waves → detectable by e.g. LISA?
- Important quantities: Nucleation temperature, transition strength, transition rate, wall speed, …
- Normally studied with perturbation theory at weak coupling…

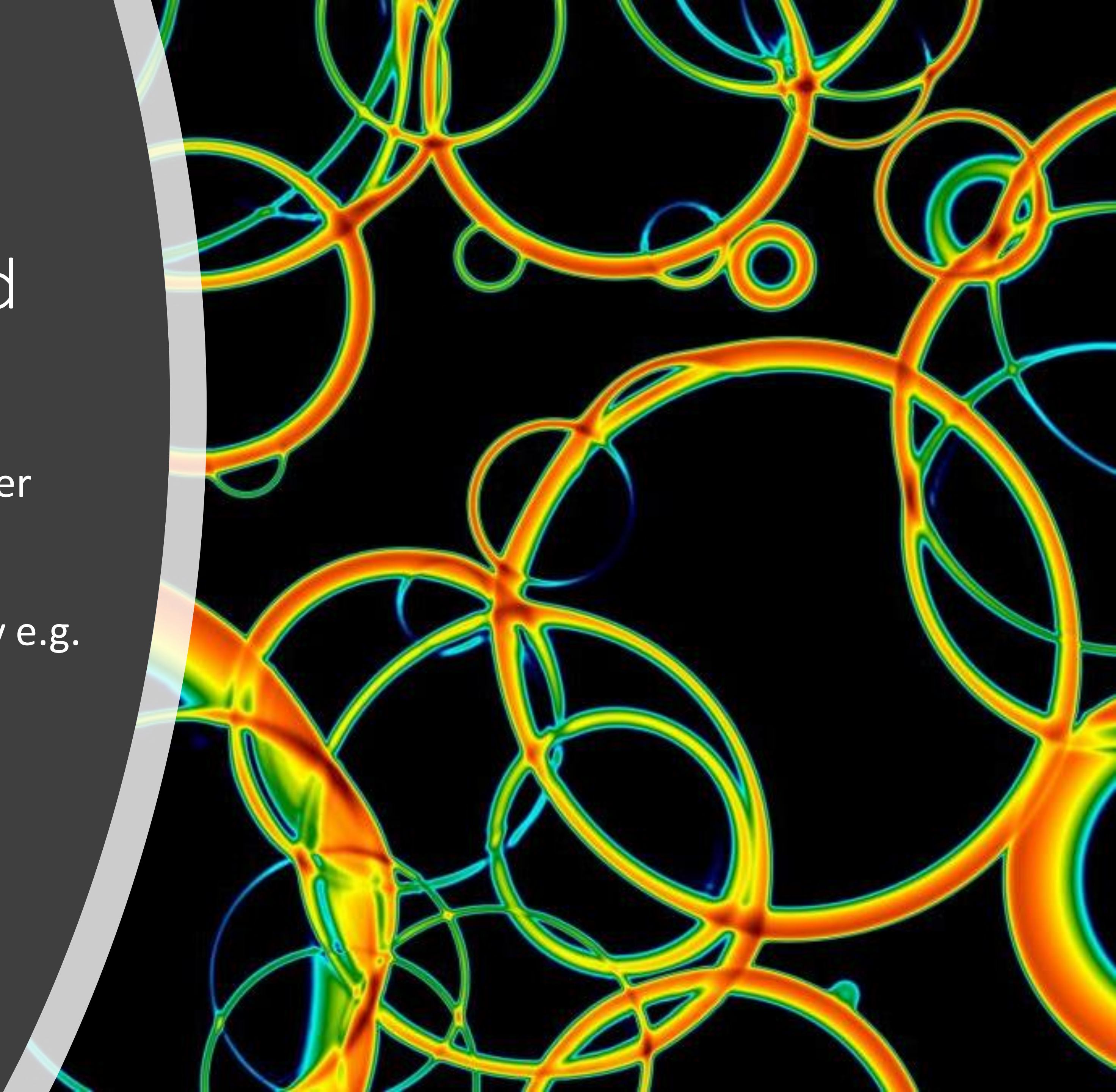
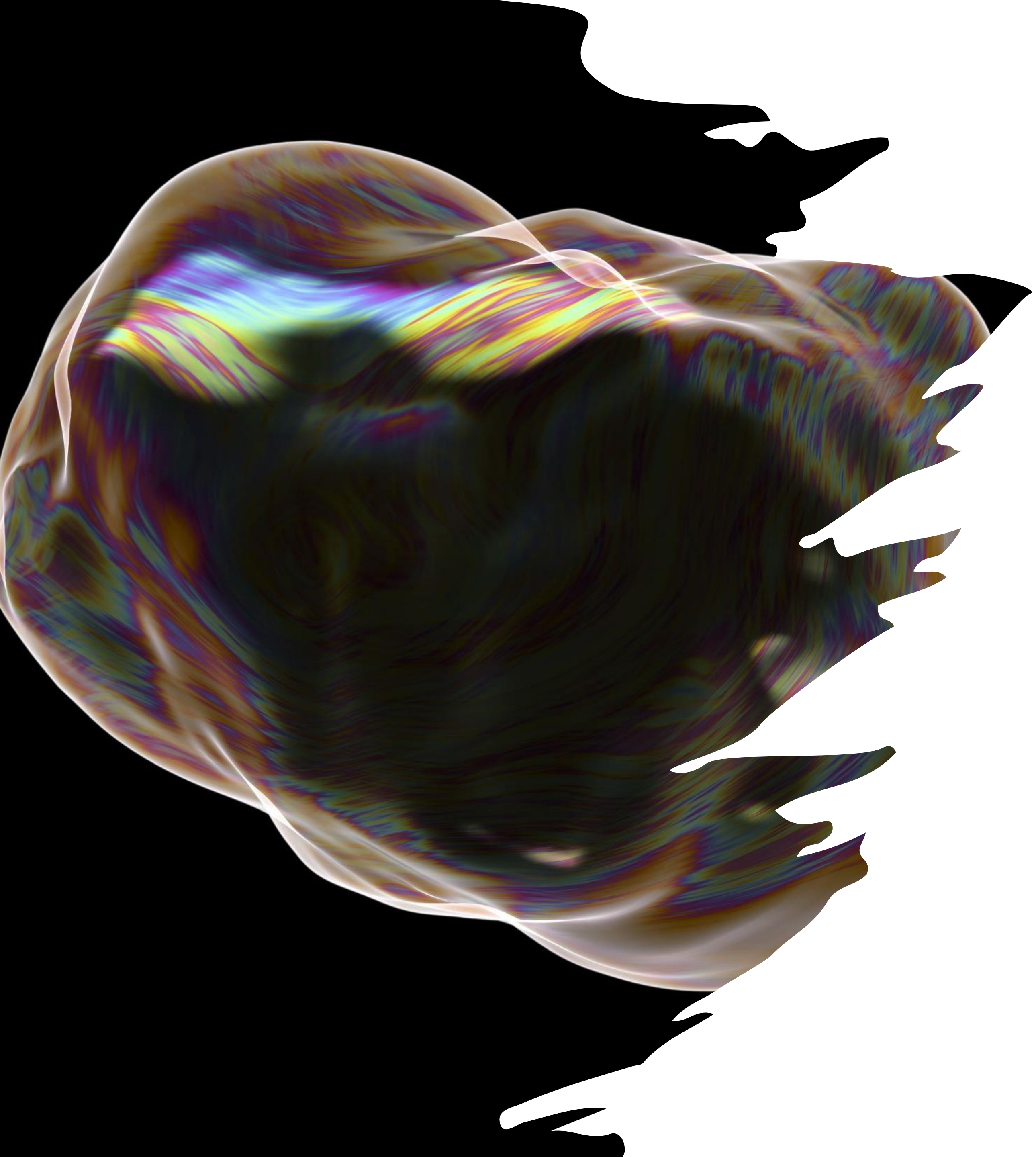


Figure by David Weir



Outline

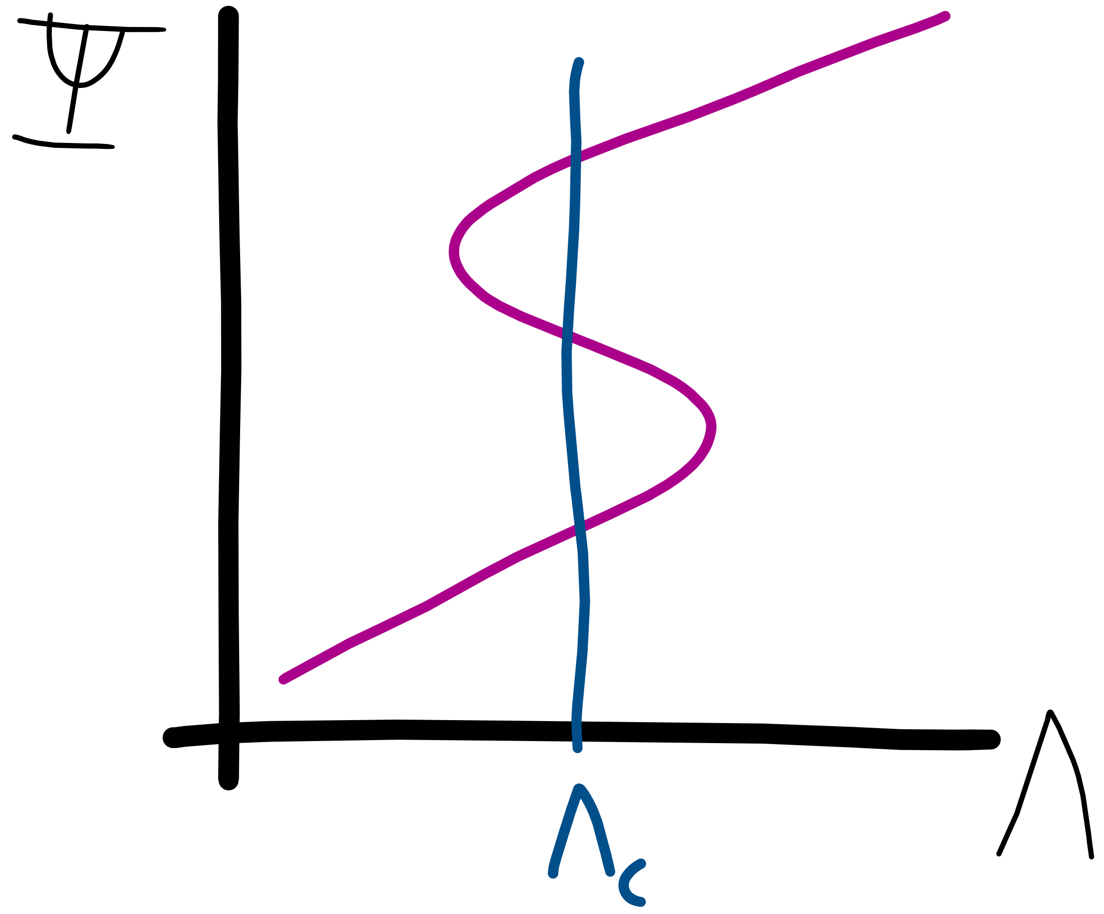
1. Basics of first order PTs and bubble nucleation
2. Holographic approach, with a simple example
3. Alternatives to bubble nucleation

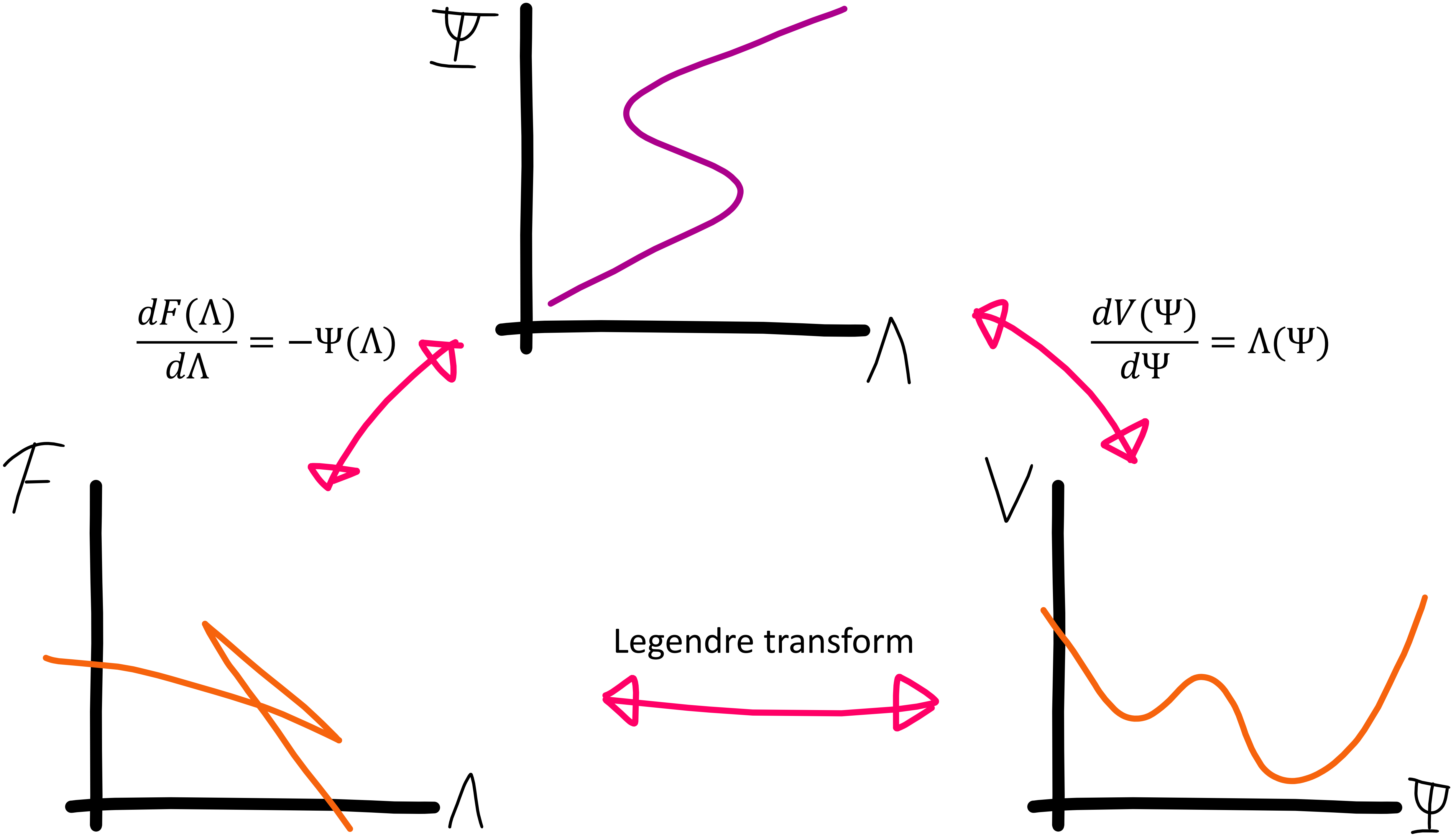
Based on…

…**2109.13784** and **2110.14442** with Fëanor Reuben Ares, Mark Hindmarsh, Carlos Hoyos & Niko Jokela.

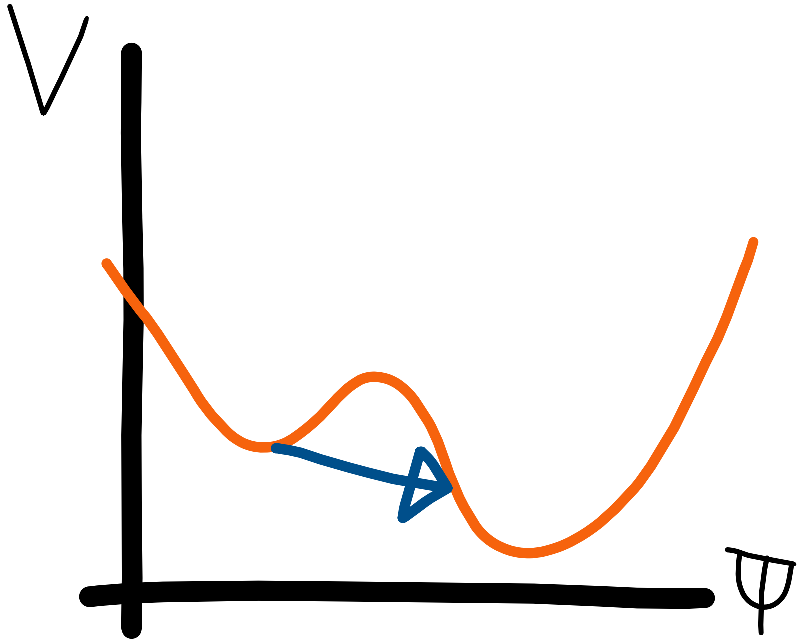
…work in progress, also with Alessio Caddeo, Xin Li, Mikel Sanchez-Garitaonandia.

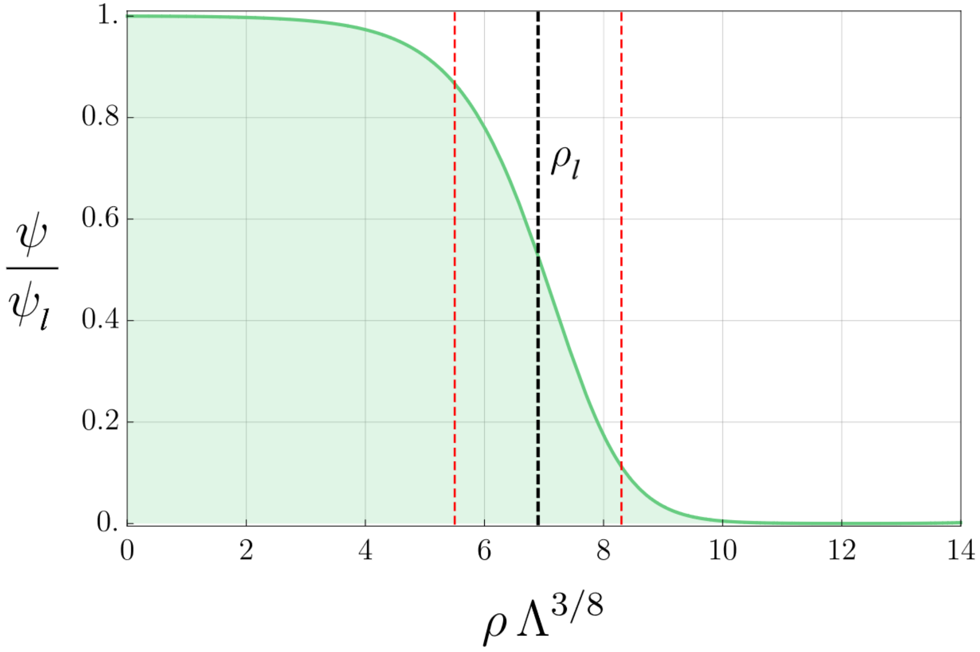
First order phase transitions – some basics




Ψ
Λ
$\frac{dF(\Lambda)}{d\Lambda} = -\Psi(\Lambda)$
$\frac{dV(\Psi)}{d\Psi} = \Lambda(\Psi)$
F
Λ
Legendre transform
V
Ψ

…typically proceed through **bubble nucleation**





Later: Other possibilities…

Bubble nucleation in QFT

- Studied by Callan, Coleman (1977), at non-zero temperature by Linde
- Transition mediated by Euclidean bubble solution
- Nucleation rate $\sim e^{-S(\text{bubble})}$

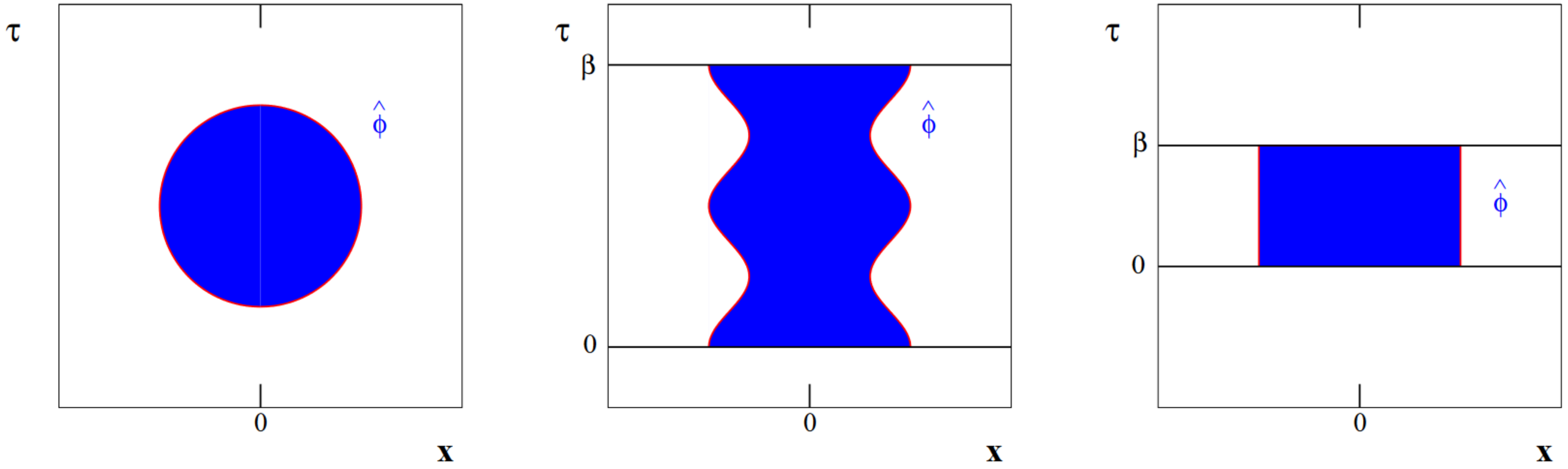
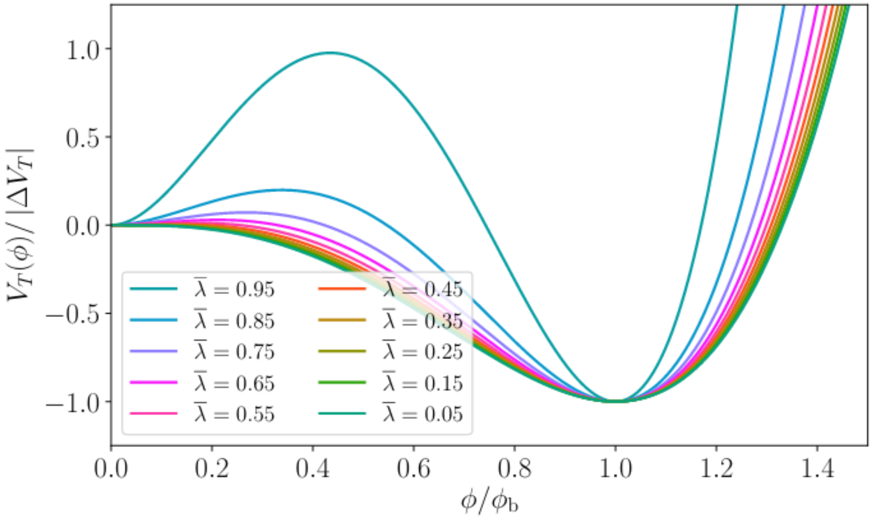
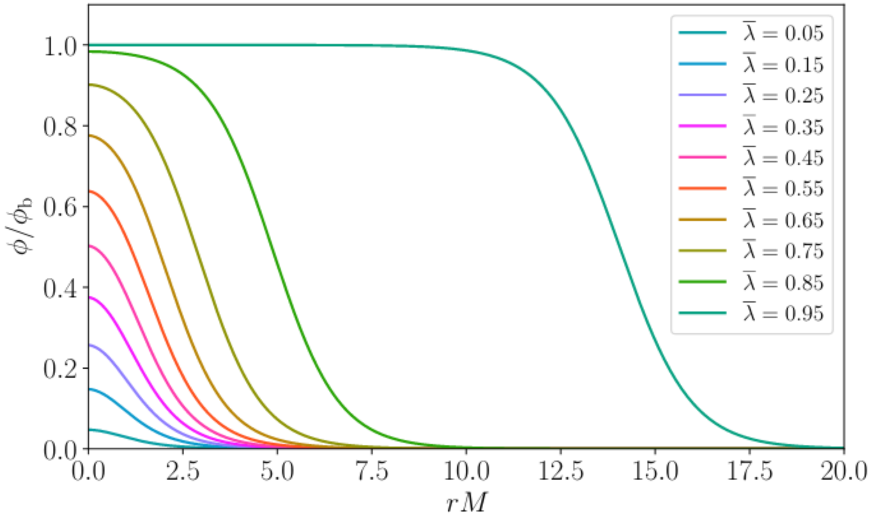


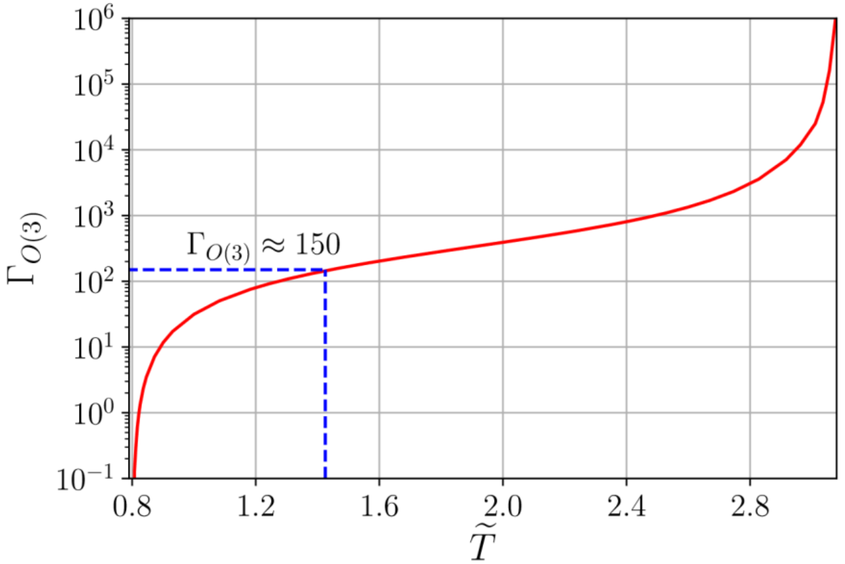
Fig. from Laine & Vuorinen '17

Thermal bubble nucleation in QFT

Figure from Hindmarsh et al. (2021)







1. For each T, find effective action
2. Solve for O(3) bubble
3. Get nucleation rate as function of T

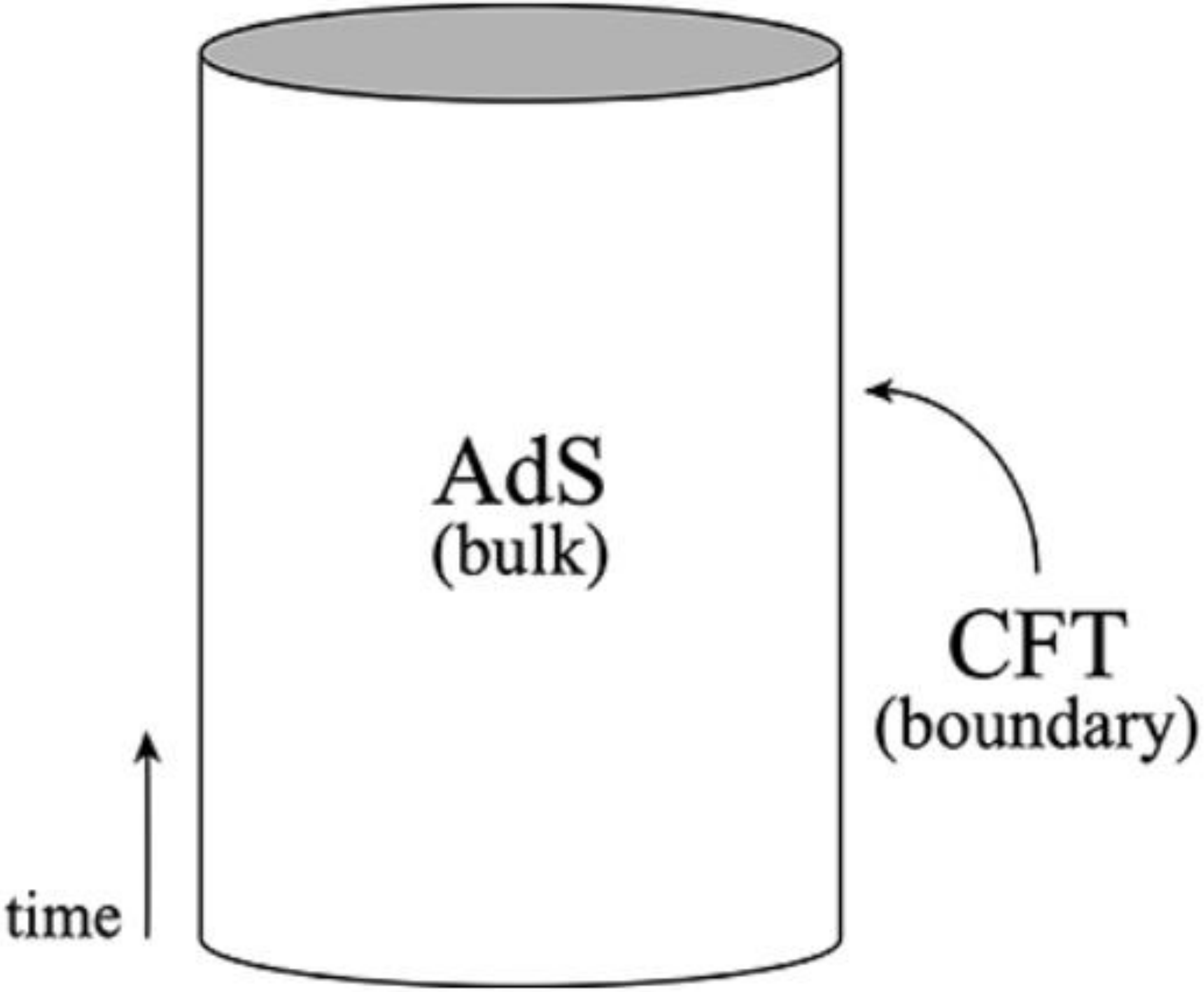
Gravity Sagas

Where’s gravity?!?

We want to study bubble nucleation…

…at strong coupling…

…using holography!



Bubbles in holography

Can look for bubble solutions directly in gravity theory

→Complicated PDEs… ☹

Easier way: compute field theory effective action for ”order parameter”

$$\Gamma[\Psi] = W[\Lambda] - \int d^4x \, \Lambda\Psi$$

→Find bubble solutions – now ODE! ☺

The **general** approach

We want to compute the effective action, in a derivative expansion…

$$\Gamma[\Psi] = -N^2 \int d^4x \left\{ V(\Psi) + \frac{1}{2} Z(\Psi)(\nabla\Psi)^2 + \cdots \right\}$$

…using holography.

- Potential $V(\Psi)$ obtained from homogeneous black brane solutions
  - → Extract S-curve $\Lambda(\Psi)$; integrate
- Kinetic term $Z(\Psi)$ obtained by fluctuations around homogeneous solutions
  - → $\Gamma[\Psi]$ generates 1PI n-pt functions
  - → 2-pt function to order k^2 gives $Z(\Psi)$

Example in a simple **toy** model

Bottom-up 5D gravity-scalar theory:

$$S = \frac{1}{2}\int \mathrm{d}^5x\,\sqrt{-g}\left\{R - \left(\partial_\mu\phi\right)^2 + \frac{12}{L^2} - m^2\phi^2\right\}$$

Choose m^2 such that **dimension of dual operator $\boldsymbol{\Psi}$ is 4/3**

Multi-trace deformations

The dual operator Ψ is a **dimension 4/3 single trace operator**.

We can deform the original CFT by ***multi-trace*** operators:

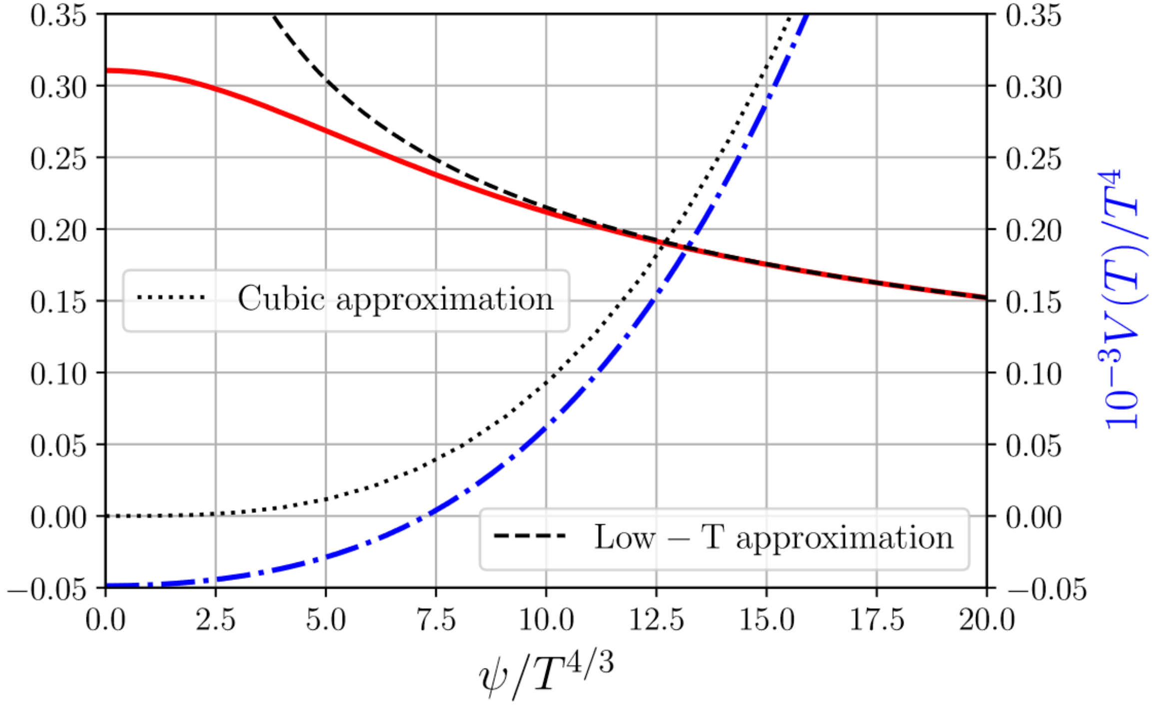
$$S_{CFT} \rightarrow S_{CFT} + \int d^4x \left\{ \Lambda\Psi + \frac{f}{2}\Psi^2 + \frac{g}{3}\Psi^3 \right\}$$

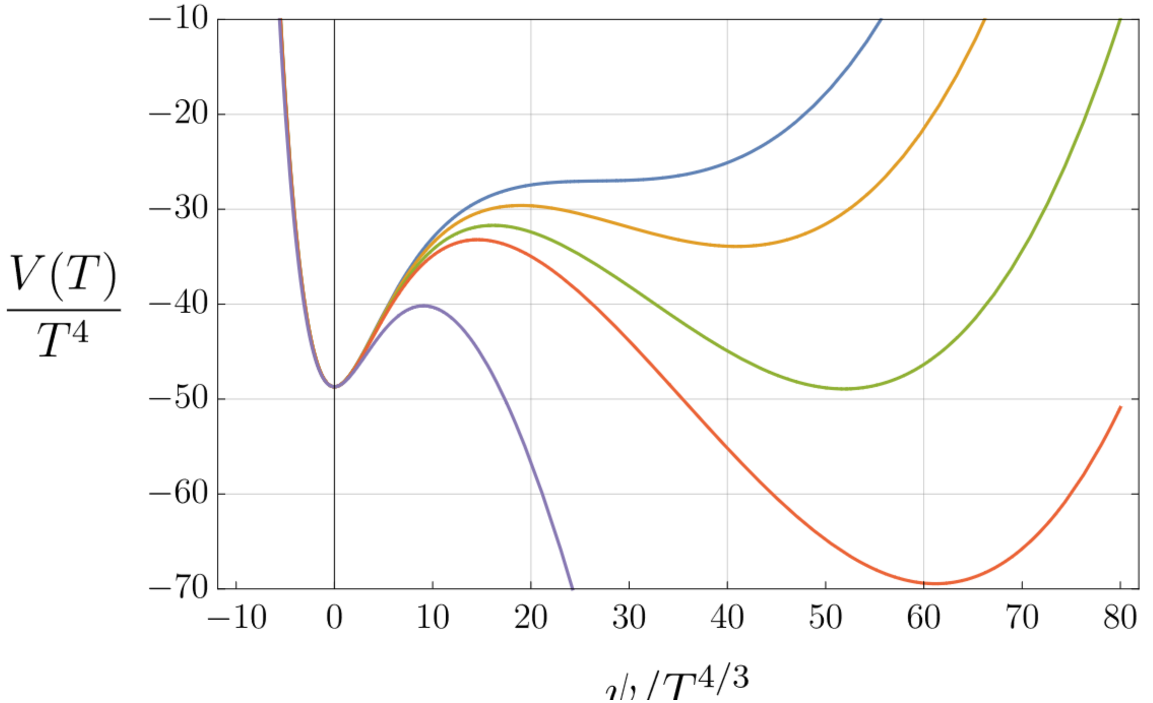
- Easy in holography: changing boundary conditions in AdS
- Easy in field theory: Simple modification of effective action

$$\Gamma[\Psi] \rightarrow \Gamma[\Psi] + \Lambda\Psi + \frac{f}{2}\Psi^2 + \frac{g}{3}\Psi^3$$

Dim-4/3 theory – results

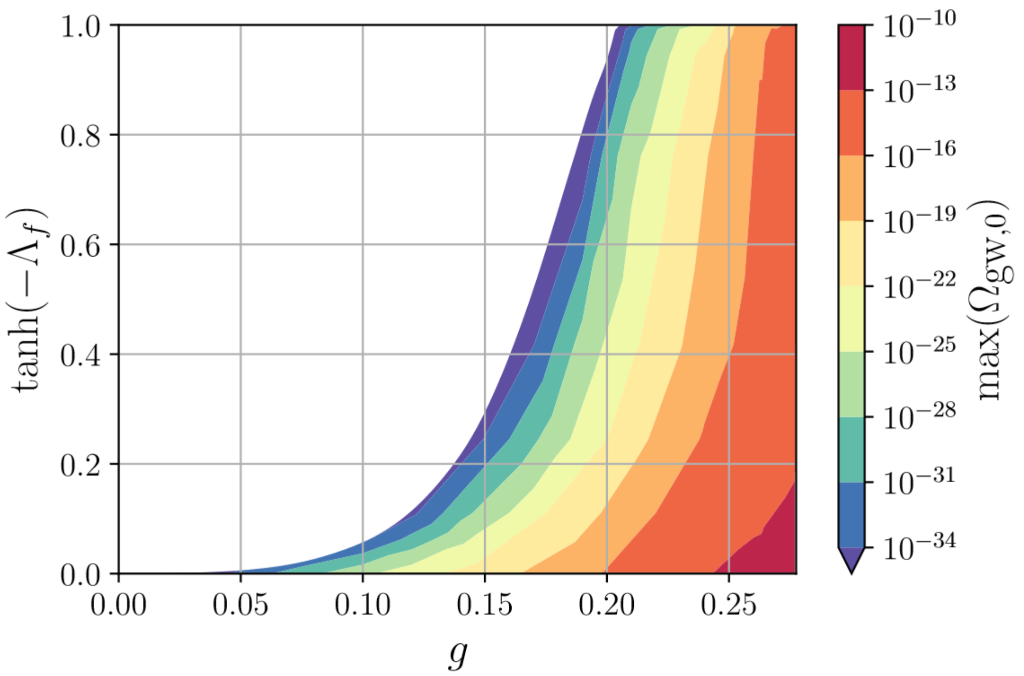
Multi-trace deformations provide ***knobs*** which can induce a first order PT!

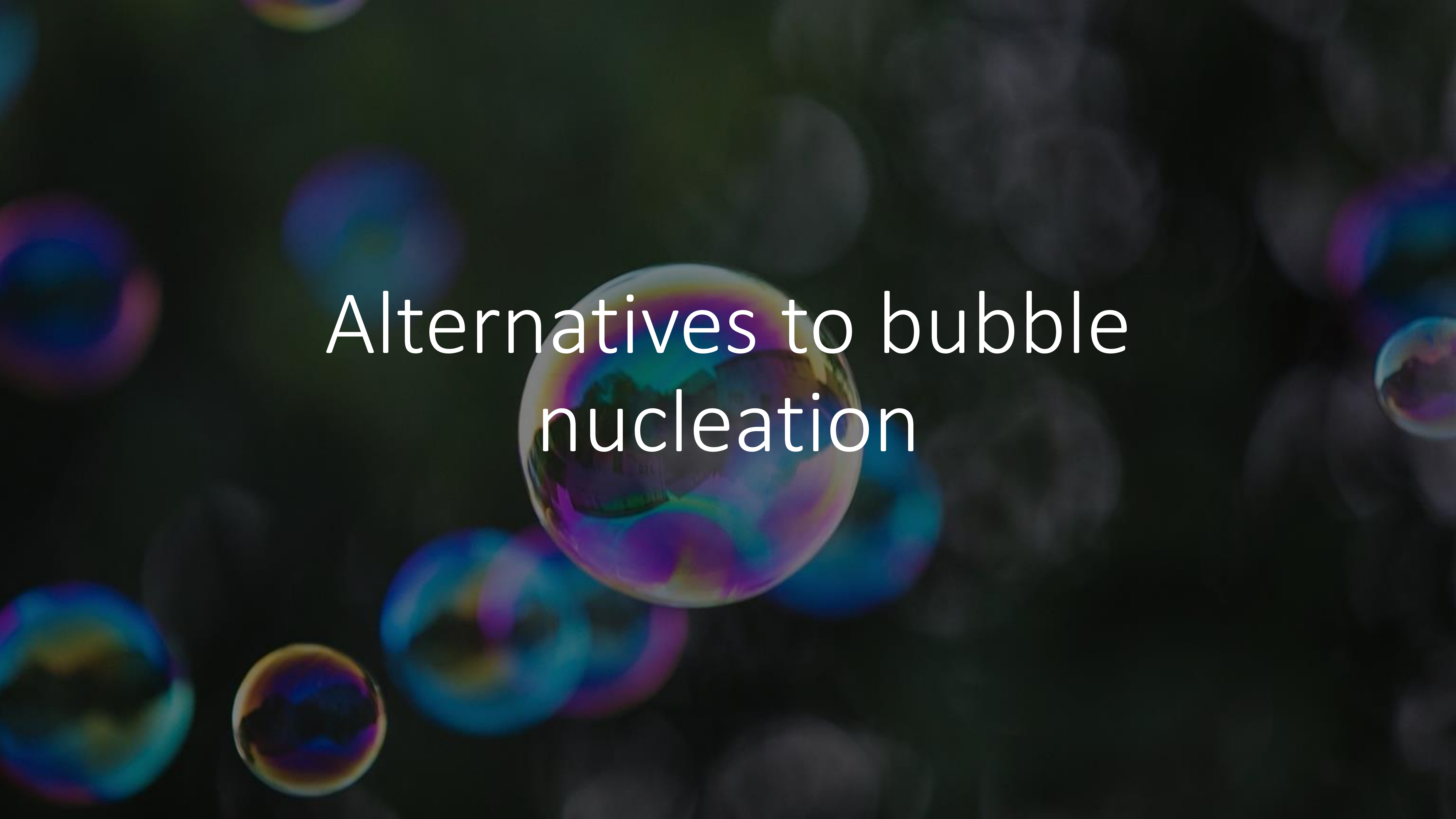




Gravitational waves!

- In our model, we compute all quasi-equilibrium GW parameters
- Use phenomenological relationship (from other holographic works) to estimate wall speed
- Use (improved) LISA cosmology working group model to find GW power spectrum →
- Detectable signal for small Λ and "large" g



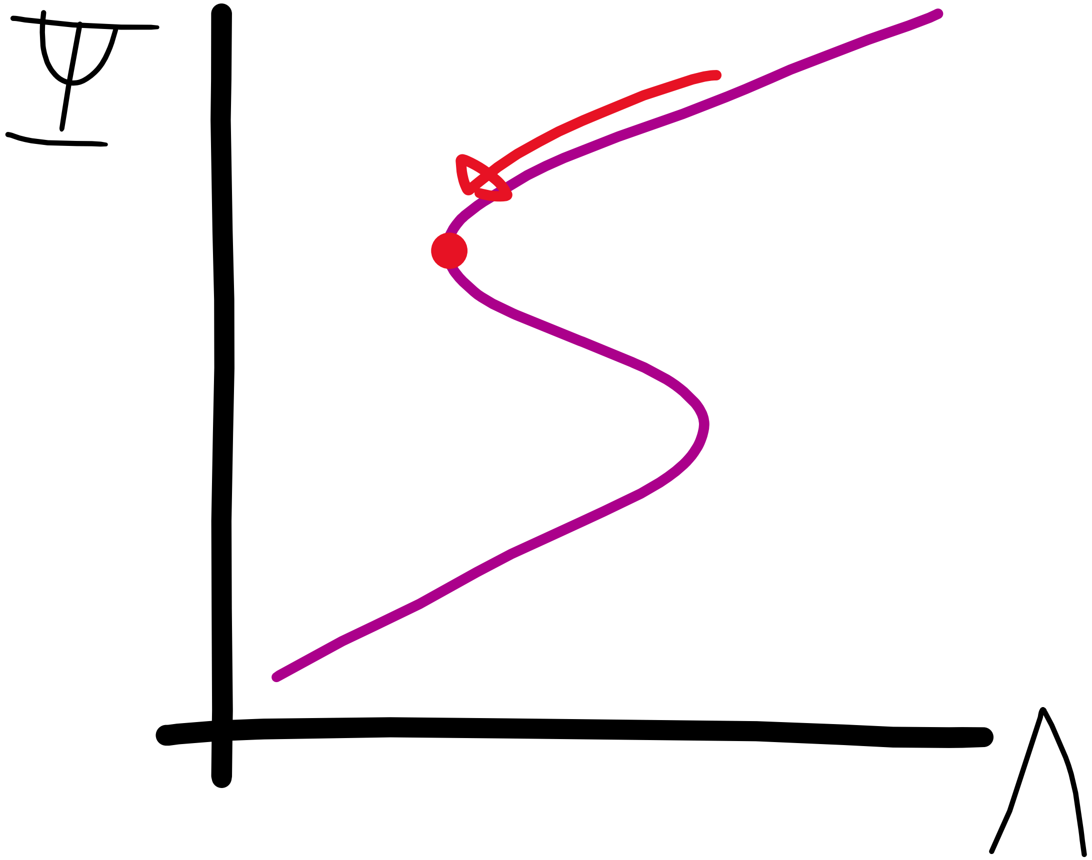


Alternatives to bubble nucleation

Other ways to transition

If bubble nucleation suppressed
(as is the case for $N \to \infty$)

→ reach spinodal point

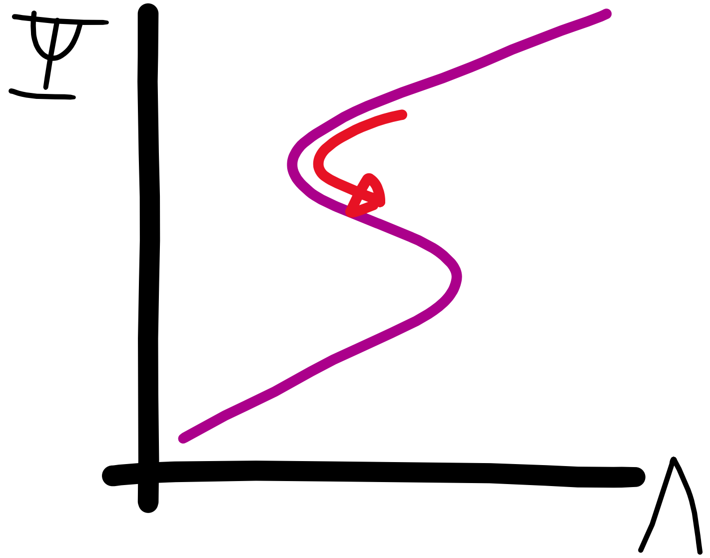


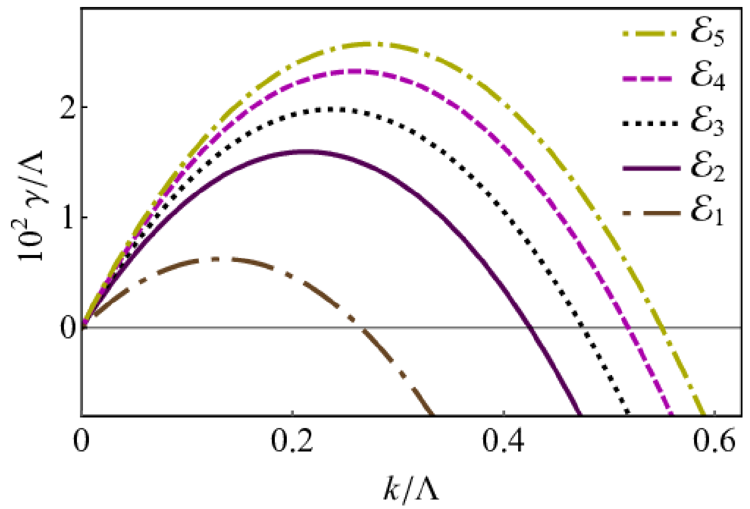
...then what?

Option 1: Spinodal decomposition

- As energy is removed, system enters unstable branch
- Temperature will start to increase
- Field perched on local maximum of effective potential
- Long-wavelength instability

See e.g. Attems et al. 1905.12544



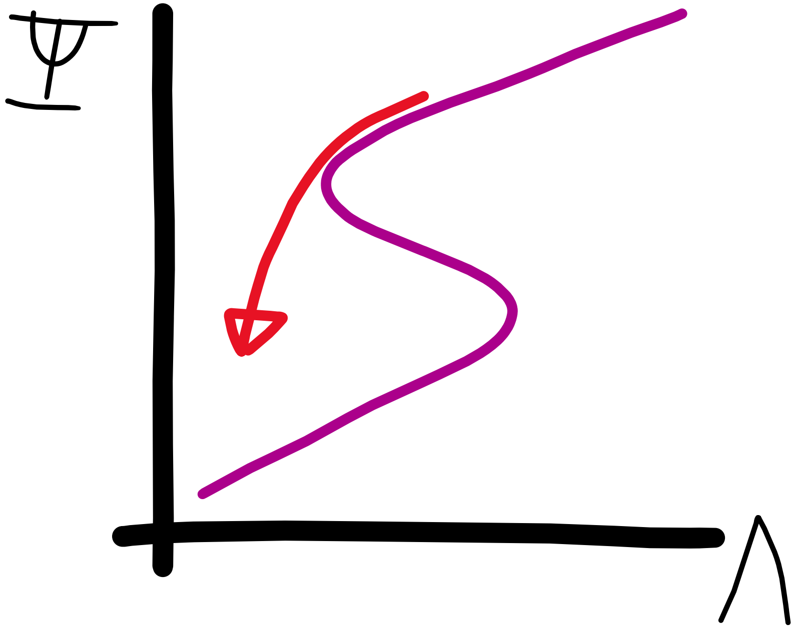


Option 2: “Forced cooling”

What if system is ***forced*** to continue cooling down?

E.g. when “PT sector” is in equilibrium with larger system which cools down.

→Effective potential “tilts over”, field rolls down to true vacuum



Not discussed in the literature?

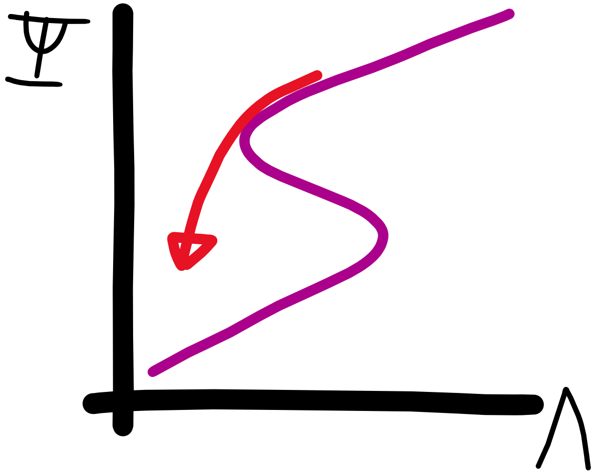
Option 2: "Forced cooling"

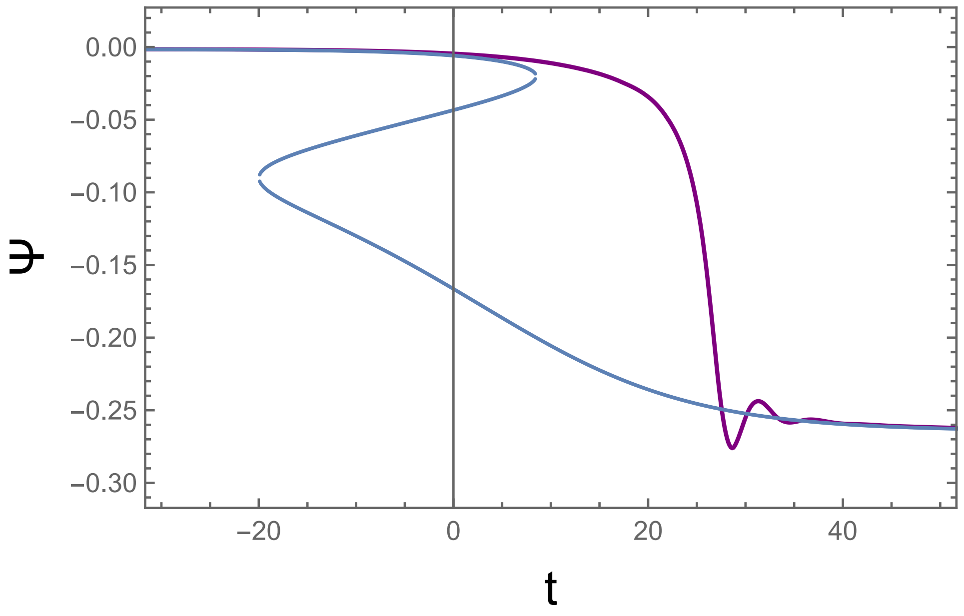
What if system is ***forced*** to continue cooling down?

E.g. when "PT sector" is in equilibrium with larger system which cools down.

→Can implement this in holography with similar multitrace model as earlier, in *probe limit*

→Preliminary results…





Summary

- 1st order PTs appear in all branches of physics (in particular: gravity, cosmology…)

- Typically proceed through bubble nucleation
  →In holography, can treat "easily" (no PDEs) by computing QFT effective action in derivative expansion

- Alternatively, one can transition through…
  →Spinodal decomposition (old)
  →"Forced cooling" (new?)

In the works…

- Compute bubble solutions on gravity side; compare with effective action
- Use effective action approach to find other non-trivial solutions?
- "Easy" method to compute wall speeds from holography? (Also: Bea et al. '21, Bigazzi et al. '21, Henriksson '22, Janik et al. '22)
- Alternative ways to complete transition → results → useful/relevant for anything???